Investments and Portfolio Theory *Final Exam*
10 July 2007

Your Name:____

Student Number: ____

Signature:____

This is a closed-book exam. You are allowed to use a calculator and a dictionary. The total time is 3 hours. There are 24 multiple choice questions (with equal weight). Circle **only** one choice for each question. There is no penalty for the wrong choice. Please sign before you start.

Hint: some problems may look familiar but are in fact different from the exercises.

Part II (12 questions)

1. The current term structure of interest rates r_t (or spot rates) implies that r_1 is 4%, r_2 is 5%, and r_3 is 4.5% (annually compounded). A private equity fund promises to pay $100,000 one year from now, $150,000 in two years, and then a final third payment of $900,000 in 3 years (end of year 3). Assuming no default risk, what is the fair value of this fund?

   a. less than $500,000

   b. between $800,000 and $860,000

   c. between $750,000 and $800,000

   d. between $860,000 and $1,020,000

   *e. more than $1,020,000

   Answer: $100/1.04 + 150/1.05^2 + 900/1.045^3 = 1020.9$

2. The one-year spot rate (semi-annual compounded) is 6%. The forward rates (semi-annually compounded) for year 2 and year 3 are ${}_1f_1 = 8\%$ and ${}_2f_1 = 10\%$ respectively. Which statement is most correct?

   a. The 1-year and 2-year annually-compounded spot rates are 6.09% and 7% respectively.

   b. The 2-year and 3-year annually-compounded spot rates are 7.05% and 8.05% respectively.

   *c. The 2-year and 3-year annually-compounded spot rates are 7.12% and 8.15% respectively.

   d. The 1-year and 3-year annually-compounded spot rates are 6.09% and 8.05% respectively.

   e. The 1-year and 2-year annually-compounded spot rates are 6% and 7.05% respectively.

   Answer: The one-year annually-compounded spot rate is 6.09% : $1.03^2 = 1.0609$.

   To get the 2-year spot rate, we have $1.03^2 * 1.04^2 = (1 + r_2)^2$, Solution is : $r_2 = 0.0712$.

   For the 3-year spot rate, we have $1.0712^2 * 1.05^2 = (1 + r_3)^3$, Solution is : $r_3 = 0.0815$.

3. Assume that the 1–year T-bill is selling at $95.6 (with a face value of $100). A 2-year 6% coupon bond (also risk free) with annual coupon payment has a yield to maturity of 4.2%. If there are no arbitrage opportunities, the price of a risk-free 7% annual coupon bond with 2 years to maturity is (choose the one closest to your solution)

   a. $102.25

   *b. $105.26

c. \$103.54

d. \$101.82

e. none of the above.

Answer: $(6 * 0.956 + 106 * x) = 6/1.042 + 106/1.042^2$, Solution is: $\{x = 0.921\,22\}$

$0.921\,22 * 107 + 0.956 * 7 = 105.\,26$

4. My pension plan will pay me \$250,000 once a year for a 3-year period. The first payment starts at exactly 10 years from now. The current term structure is flat and the interest rate is 5%, annually compounded. The pension fund wants to immunize its position by duration matching using the 5-year and 15-year zero-coupon bonds. The hedging strategy requires investing

   *a. higher than \$250,000 in the 15-year zero-coupon bond.

   b. between \$200,000 and \$250,000 in the 15-year zero-coupon bond.

   c. less than \$150,000 in the 5-year zero-coupon bond.

   d. higher than \$200,000 in the 5-year zero-coupon bond.

   e. none of the above.

   Answer: We need to derive the present value and the duration of this liability first.

   $PV = 250000(\frac{1}{1.05^{10}} + \frac{1}{1.05^{11}} + \frac{1}{1.05^{12}}) = 4.\,388\,6 \times 10^5$

   $\text{Duration} = \frac{250000}{4.\,388\,6 \times 10^5}(\frac{10}{1.05^{10}} + \frac{11}{1.05^{11}} + \frac{12}{1.05^{12}}) = 10.\,967$

   $x * 5 + (1 - x) * 15 = 10.9617$, Solution is: $0.403\,83$

   In 5-year zero: $0.403\,83 * 4.\,388\,6 \times 10^5 = 1.\,772\,2 \times 10^5$

   In 15-year zero: $(1 - 0.403\,83) * 4.\,388\,6 \times 10^5 = 2.\,616\,4 \times 10^5$

5. The Chicago Board of Trade has just introduced a new futures contract on XYZ stock, a company that currently pays no dividends. Each contract calls for delivery of 1,000 shares of stock in one year. The XYZ stock now sells at \$120 per share. The T-bill rate is 5% per year. The margin on the contract is \$12,000. If XYZ's price suddenly increases by 6%, then the percentage return on an investor's position (ROE) who holds a long futures contract on XYZ and a margin account of \$12,000 will be

   a) 60%.

   b) −60%.

   *c) 63%.

   d) −63%.

   e) none of the above.

Answer: $F = 120(1.05) = 126.0$. $F1 = 120 * 1.06(1.05) = 133.56$. $F1 - F = 133.56 - 126 = 7.56$.

Thus $ROE = 7.56 * 1000/12000 = 0.63$.

This is for the next two questions. Consider a binomial model with two states. A stock ABC is selling at $S_0 = \$100$ per share. The risk-free rate is 10%. The two possibilities at time 1 for the stock price S_1 are \$130 and \$80. There are call and put options traded on ABC and for simplicity assume that one option is on one share of the stock.

6. Which of the following is your best choice? In the absence of arbitrage, a call option on ABC with strike price equal to \$110

   a) has a price equal to \$10.91.

   b) has a hedge ratio of -0.6.

   c) has a hedge ratio of 0.4.

   *d) both a) and c).

   e) none of the above.

   Answer: $h = (C^+ - C^-)/(S^+ - S^-) = 20/50 = 0.4$.

   $G = C - 0.4S = (0 - 0.4 * 80)/1.1 = -29.091 \Rightarrow C = G + 0.4S = -29.091 + 0.4 * 100 = 10.909$.

   Checking the price with risk-neutral probability $q = (R - D)/(U - D) = (1.1 - 0.8)/(1.3 - 0.8) = 0.6$

   $C = 0.6 * 20/1.1 = 10.909$

7. Suppose you hold 100 shares of ABC and wish to use put options on ABC to temporarily hedge the stock's price risk. How many put options P with strike price equal to \$110 should you buy (or sell)?

   a. you should sell about 60 contracts of P.

   b. you should sell about 167 contracts of P.

   c. you should buy about 600 contracts of P.

   *d. you should buy about 167 contracts of P.

   e. you should sell about 600 contracts of P.

   Answer: Let $G = S - hP$

   $h = \frac{S^+ - S^-}{P^+ - P^-} = \frac{50}{0-30} = -1.6667$. A negative hedge ratio indicates buying the puts. The number to buy is $1.6667 * 100 = 166.67$.

   Verifying: $G^+ = 100 * 130 = 13000.0$

   $G^- = 100 * 80 + 166.67 * 30 = 13000.0$

8. You bought today a one-year US dollar futures contract at 1.02 euros per dollar and placed a margin of 15,000 euros. The contract size is $200,000. At the close of the day the *spot* exchange rate is 1.015 euros per dollar, and the euro and dollar one-year (annually compounded) interest rates are 4% and 3% respectively. Assuming no arbitrage, your return on equity (ROE) satisfies

a. $10\% \leq ROE$.

*b. $5\% \leq ROE < 10\%$.

c. $-5\% \leq ROE < 5\%$.

d. $-7\% \leq ROE < -5\%$.

e. $ROE < -10\%$.

Answer: The futures price becomes $F = 1.015 * 1.04/1.03 = 1.0248$

Thus your return on equity is $(1.0248 - 1.02) \times 200000/15000 = 0.064$.

9. Yields on short-term bonds tend to be more volatile than yields on long-term bonds. Suppose that you have estimated that the yield on 30-year bonds changes by 10 basis points for every 15-basis-point move in the yield on 5-year bonds. You hold a $1 million portfolio of 5-year maturity bonds with modified duration 4 years. You desire to hedge your interest rate exposure with 30-year T-bond futures, which has $100,000 par value, a modified duration of 7.9 years, and sells at $F = \$95,000$.

hoeft niet (duration)

a) You should buy 7 futures contracts.

b) You should buy 8 futures contracts.

*c) You should sell 8 futures contracts.

d) You should sell 7 futures contracts.

e) You should buy 12 futures contracts.

Answer: $dB_1 = -B_1 * D_1 * dr_1$, $dB_2 = -B_2 * D_2 * dr_2 = -B_2 * D_2 * dr_1/1.5$.
$h = dB_1/dB_2 = 1.5 * B_1 * D_1/(B_2 * D_2) = 1.5 * 1000 * 4/(95 * 7.9) = 7.9947$

10. A stock is priced at $30 today and will pay a dividend of $0.5 per share tomorrow. No more dividend is expected thereafter for the coming year. The call and put options on this stock (of European type) with the same exercise price of $28, expiration date 1 year from now, are currently traded at $4.5 and $2.0 respectively. The risk-free interest rate is 6% per annum. Which of the following best describes an arbitrage strategy?

*a. Buy the call, sell the stock, pay dividend, save cash, and sell the put.

b. Buy the call, sell the stock, pay dividend, borrow cash, and buy the put.

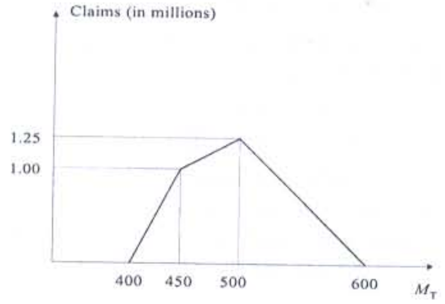

Figure 1: Contingent customer claims for damage.

c. Sell the call, buy the stock, receive dividend, borrow cash, and buy the put.

d. Sell the call, buy the stock, receive dividend, borrow cash, and sell the put.

e. Borrow cash to buy the stock, hedge with a long put and a short call, and receive dividend.

Answer:

$$30.9 = C + X(1.06)^{-1} < S + P - D = 31.5$$

This suggests:

Transactions	$t=0$	$t=1$ $(S>28)$	$t=1$ $(S \leq 28)$
Buy Call	-4.5	S-28	0
Save Cash	- 28/1.06	28	28
Sell Share	30	-S	-S
Sell Put	2.0	0	-(28-S)
Pay Dividend	-0.5	0	0
Total	0.6	0	0

11. The actuary of an insurance company anticipates that customer claims for damage in $T = 6$ months time will depend as follows on the level of the market index M_T at that time:

There is a market for 6-month call and put options on the index with strike prices X of 400, 450, 500, 550 and 600; each of these options has as underlying asset $\$100 \times M$. Which of the following option positions will provide a perfect hedge of the contingent claims?

(a) 200 long calls with $X = 400$; 150 short calls with $X = 450$; 175 short calls with $X = 500$; and 125 short calls with $X = 600$.

(b) 200 short calls with $X = 400$; 150 long calls with $X = 450$; 175 long calls with $X = 500$; and 125 long calls with $X = 600$.

(c) 125 short puts with $X = 600$, 175 short puts with $X = 500$; 150 short puts with $X = 450$; and 200 long puts with $X = 400$.

(d) 50 short puts and 125 short calls with $X = 500$; 150 short puts with $X = 450$; 200 long puts with $X = 400$; and 125 long calls with $X = 600$. The position also involves a time deposit of cash equal to the present value of 1.25 million dollars.

(e) None of the above.

Just try to verify that (d) is correct. All rests are wrong.

12. Which of the following statements about Turbo is *wrong*?

a. A turbo-short is similar to selling short the underlying asset and deposit the proceeds in the bank to earn interests.

b. A turbo's price can be affected by its financial level as well as the underlying asset's price.

c. Since turbo involves leverage, investing in a turbo could incur losses up to 100% of your investment despite the stop-loss levels.

*d. The financial level of a turbo-short is always lower than its stop-loss level, although it is not true for the turbo-long.

e. The financial level of a turbo on a stock typically goes up day by day unless the stock pays a dividend.

Answer: The financial level of a turbo-short is always higher than its stop-loss level, or else the bank may suffer severe losses. The other statements are true by definition (see slides week 13).