UITWERKING TENTAMEN BASIS ACTUARIAAT 2

16 juni 2006

14.00-17.00 uur

Opgave 1

$$
\begin{aligned}
a_{x:\overline{n}|} &= \sum_{k=1}^{n-1} a_{\overline{k}|}\, {}_k p_x\, q_{x+k} + a_{\overline{n}|}\, {}_n p_x \\
&= \sum_{k=1}^{n-1} \sum_{j=1}^{k} v^j\, {}_k p_x\, q_{x+k} + a_{\overline{n}|}\, {}_n p_x \\
&= \sum_{j=1}^{n-1} \sum_{k=j}^{n-1} v^j\, {}_k p_x\, q_{x+k} + a_{\overline{n}|}\, {}_n p_x \\
&= \sum_{j=1}^{n-1} v^j ({}_j p_x - {}_n p_x) + a_{\overline{n}|}\, {}_n p_x \\
&= \sum_{j=1}^{n} v^j ({}_j p_x - {}_n p_x) + \sum_{j=1}^{n} v^j\, {}_n p_x \\
&= \sum_{j=1}^{n} v^j\, {}_j p_x
\end{aligned}
$$

Opgave 2

$$
Y_1 = \begin{cases} a_{\overline{K}|}, & K < n \\ a_{\overline{n}|}, & K \geq n \end{cases}
$$

$$
Y_2 = \begin{cases} 0, & K < n \\ v^n a_{\overline{K-n}|}, & K \geq n \end{cases}
$$

$$
Y_1 Y_2 = \begin{cases} 0, & K < n \\ a_{\overline{n}|} v^n a_{\overline{K-n}|}, & K \geq n \end{cases}
$$

$$
\begin{aligned}
\mathrm{Cov}[Y_1, Y_2] &= E[Y_1 Y_2] - E[Y_1] E[Y_2] \\
&= a_{\overline{n}|}\, {}_{n|}a_x - a_{x:\overline{n}|}\, {}_{n|}a_x \\
&> 0
\end{aligned}
$$

Opgave 3

$$\ddot{a}^{(12)}_{x:\overline{1/2|}} = \frac{1}{12}\sum_{k=0}^{5} v^{k/12}\, {}_{k/12}p_x$$
$$= \frac{1}{12}\sum_{k=0}^{5} v^{k/12}(1 - {}_{k/12}q_x)$$
$$= \frac{1}{12}\sum_{k=0}^{5} v^{k/12}\left(1 - \frac{k}{12}q_x\right)$$
$$= \frac{1}{12}\sum_{k=0}^{5} v^{k/12} - \frac{1}{12}q_x \sum_{k=0}^{5} v^{k/12}\frac{k}{12}$$
$$= \ddot{a}^{(12)}_{\overline{1/2|}} - \frac{1}{12}q_x\,(I^{(12)}a)_{\overline{5/12|}}$$

Opgave 4

a. $$L = \begin{cases} v^{K+1}v^{n-K-1} - P\ddot{a}_{\overline{K+1|}}, & K < m \\ v^{K+1}v^{n-K-1} - P\ddot{a}_{\overline{m|}}, & m \le K < n \\ v^{n} - P\ddot{a}_{\overline{m|}}, & K \ge n \end{cases}$$

oftewel

$$L = \begin{cases} v^{n} - P\ddot{a}_{\overline{K+1|}}, & K < m \\ v^{n} - P\ddot{a}_{\overline{m|}}, & K \ge m \end{cases}$$

b. Nul stellen van E[L] levert

$$P = \frac{v^n}{\ddot{a}_{x:\overline{m|}}}.$$

c. Er geldt dat

$$L = \begin{cases} \frac{P}{d}v^{K+1} + v^{n} - \frac{P}{d}, & K < m \\ \frac{P}{d}v^{m} + v^{n} - \frac{P}{d}, & K \ge m \end{cases}$$

Verder hebben we

$$L_g = \begin{cases} v^{K+1} - P_g\ddot{a}_{\overline{K+1|}} = \left(1 + \frac{P_g}{d}\right)v^{K+1} - \frac{P_g}{d}, & K < m \\ v^{m} - P_g\ddot{a}_{\overline{m|}} = \left(1 + \frac{P_g}{d}\right)v^{m} - \frac{P_g}{d}, & K \ge m \end{cases}$$

Zij Z de stochast voor de contante waarde van de uitkering voor een gemengde verzekering. Dan geldt dat

$$\mathrm{Var}[L] = \left(\frac{P}{d}\right)^2 \mathrm{Var}[Z],$$

en dat

$$\mathrm{Var}[L_g] = \left(1 + \frac{P_g}{d}\right)^2 \mathrm{Var}[Z].$$

Dus

$$\begin{aligned}
\text{Var}[L] &= \left(\frac{P}{d}\right)^2 \left(\frac{d}{d+P_g}\right)^2 \text{Var}[L_g] \\
&= \left(\frac{P}{d+P_g}\right)^2 \text{Var}[L_g] \\
&= \left(\frac{dv^n}{1-A_{x:\overline{m}|}}\right)^2 \left(\frac{1}{d+\frac{A_{x:\overline{m}|}}{\frac{1-A_{x:\overline{m}|}}{d}}}\right)^2 \text{Var}[L_g] \\
&= \left(\frac{dv^n}{1-A_{x:\overline{m}|}}\right)^2 \left(\frac{1}{\frac{d-dA_{x:\overline{m}|}+dA_{x:\overline{m}|}}{1-A_{x:\overline{m}|}}}\right)^2 \text{Var}[L_g] \\
&= v^{2n}\text{Var}[L_g]
\end{aligned}$$

Opgave 5

Voor $k < n$ geldt dat

$${}_{k+1}V^n = \frac{1}{{}_{k+1}E_x}(P^n \ddot{a}_{x:\overline{k+1}|})$$

$${}_{k+1}V^{n+1} = \frac{1}{{}_{k+1}E_x}(P^{n+1} \ddot{a}_{x:\overline{k+1}|})$$

Aangezien $P^n > P^{n+1}$ (evident) geldt dat op dit domein dat ${}_{k+1}V^n > {}_{k+1}V^{n+1}$.

Voor $k \geq n$ geldt dat

${}_{k+1}V^n = {}_{k+1}V^{n+1} = \ddot{a}_{x+k+1}$.

Invullen in de algemene variantieformule, met $u^o_{k+1} = 0$, bewijst het gestelde.

Opgave 6

a. $Z = \begin{cases} v^{T(x)}, T(y) > T(x) \\ 0, \quad T(y) \leq T(x) \end{cases}$

b.

$$E[Z] = \int_0^\infty v^t \; {}_t p_x \, m_{x+t} \; {}_t p_y \, dt$$

$$= m_x \int_0^\infty e^{-(d + m_x + m_y)t} dt$$

$$= \frac{m_x}{d + m_x + m_y}$$

$$= \frac{0{,}05}{0{,}03 + 0{,}05 + 0{,}04}$$

$$= \frac{5}{12}$$

$$E[Z^2] = \int_0^\infty v^{2t} \; {}_t p_x \, m_{x+t} \; {}_t p_y \, dt$$

$$= m_x \int_0^\infty e^{-(2d + m_x + m_y)t} dt$$

$$= \frac{m_x}{2d + m_x + m_y}$$

$$= \frac{0{,}05}{0{,}06 + 0{,}05 + 0{,}04}$$

$$= \frac{1}{3}$$

$$\text{Var}[Z] = \frac{1}{3} - \left(\frac{5}{12}\right)^2.$$