ANTWOORDEN
TENTAMEN BASIS ACTUARIAAT 2, 9 JULI 2004

Opgave 1

Uitkeringenpatroon:

Tijdstip:	*0*	*1*	*2*	*3*	…	*n-1*	N
$(Da)_{x\,\overline{n}\rvert}$	0	n	n-1	n-2	…	2	1
$v\,p_x\,(D\ddot{a})_{x+1\,\overline{n-1}\rvert}$	0	n-1	n-2	n-3	…	1	0
$X=(Da)_{x\,\overline{n}\rvert}-v\,p_x\,(D\ddot{a})_{x+1\,\overline{n-1}\rvert}$	0	1	1	1	…	1	1

Hieruit volgt: $X = a_{x:\overline{n}\rvert}$

Opgave 2

$$(I\ddot{a})^{(p)}_{x:\overline{n}\rvert}=\sum_{k=0}^{n-1}{}_kE_x\cdot\ddot{a}^{(p)}_{x+k:\overline{n-k}\rvert}=\sum_{k=0}^{n-1}{}_kE_x\left(\ddot{a}_{x+k:\overline{n-k}\rvert}-\left(1-{}_{n-k}E_{x+k}\right)\tfrac{p-1}{2p}\right)=(I\ddot{a})_{x:\overline{n}\rvert}-\tfrac{p-1}{2p}\sum_{k=0}^{n-1}\left({}_kE_x-{}_nE_x\right)$$

$$=(I\ddot{a})_{x:\overline{n}\rvert}-\tfrac{p-1}{2p}\left(\ddot{a}_{x\overline{n}\rvert}-n\,{}_nE_x\right)$$

Opgave 3

a. $${}_0L=\begin{cases}(K+1)v^{K+1}-P\,\ddot{a}_{\overline{K+1}\rvert} & 0\le K<n\\ n\,P\,v^n\qquad-\quad P\,\ddot{a}_{\overline{n}\rvert}\end{cases}$$

b. $P\,\ddot{a}_{x:\overline{n}\rvert}=(IA)^{1}_{x:\overline{n}\rvert}+n\,P\,A_{x\,\overline{n}\rvert}^{\;\;\,1}$ => $P\,\ddot{a}_{x:\overline{n}\rvert}=\dfrac{(IA)^{1}_{x:\overline{n}\rvert}}{\ddot{a}_{x:\overline{n}\rvert}-n\,A_{x:\overline{n}\rvert}^{\;\;\,1}}$

Opgave 4

$$1-\left(A^{1}_{x+k:\overline{n-k}\rvert}-\frac{A^{1}_{x:\overline{n}\rvert}}{\ddot{a}_{x:\overline{n}\rvert}}\ddot{a}_{x+k:\overline{n-k}\rvert}\right)\;>\;A_{x+k:\overline{n-k}\rvert}^{\;\;\;\;\;\;1}-\frac{A_{x:\overline{n}\rvert}^{\;\;\,1}}{\ddot{a}_{x:\overline{n}\rvert}}\ddot{a}_{x+k:\overline{n-k}\rvert}$$

$$\Leftrightarrow\quad 1-\left(A^{1}_{x+k:\overline{n-k}\rvert}+A_{x+k:\overline{n-k}\rvert}^{\;\;\;\;\;\;1}-\frac{A^{1}_{x:\overline{n}\rvert}+A_{x:\overline{n}\rvert}^{\;\;\,1}}{\ddot{a}_{x:\overline{n}\rvert}}\ddot{a}_{x+k:\overline{n-k}\rvert}\right)\;>\;0$$

$$\Leftrightarrow\quad 1-A_{x+k:\overline{n-k}\rvert}+\frac{A_{x:\overline{n}\rvert}}{\ddot{a}_{x:\overline{n}\rvert}}\ddot{a}_{x+k:\overline{n-k}\rvert}\;>\;0\quad\Leftrightarrow\quad 1-\left(1-d\ddot{a}_{x+k:\overline{n-k}\rvert}\right)+\frac{1-d\ddot{a}_{x:\overline{n}\rvert}}{\ddot{a}_{x:\overline{n}\rvert}}\ddot{a}_{x+k:\overline{n-k}\rvert}\;>0$$

$$\Leftrightarrow\quad d\ddot{a}_{x+k:\overline{n-k}\rvert}+\left(\frac{1}{\ddot{a}_{x:\overline{n}\rvert}}-d\right)\ddot{a}_{x+k:\overline{n-k}\rvert}\;>\;0\qquad\Leftrightarrow\quad\frac{\ddot{a}_{x+k:\overline{n-k}\rvert}}{\ddot{a}_{x:\overline{n}\rvert}}>\;0\qquad QED$$

Opgave 5

$$ {}_kV^{PROSP} = \frac{i}{d}(IA)^{\,1}_{x+k:\overline{n-k}|} + k \cdot \frac{i}{d} A^{\,1}_{x+k:\overline{n-k}|} + nA_{x+k:\overline{n-k}|}^{\;\;\;\;\;\;1} - P\ddot{a}_{x+k:\overline{n-k}|} $$

Opgave 6

a. $\ddot{a}_{\overline{x:y}} - a_{x|y} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{x:y} - \left(\ddot{a}_y - \ddot{a}_{x:y}\right) = \ddot{a}_x > a_x$

Opgave 7

$$ \ddot{a}_{x|x} = \ddot{a}_x - \ddot{a}_{x:x} = \left(1 + vp_x \ddot{a}_{x+1}\right) - \left(1 + vp_{x:x}\ddot{a}_{x+1:x+1}\right) = vp_x\left(\ddot{a}_{x+1} - p_x\, \ddot{a}_{x+1:x+1}\right) $$

$$ = vp_x\left(\ddot{a}_{x+1} - \ddot{a}_{x+1:y+1} + \left(1 - p_x\right)\ddot{a}_{x+1:x+1}\right) = {}_1E_x\, \ddot{a}_{x+1|x+1} + v\, p_x\, q_x\, \ddot{a}_{x+1:x+1} $$

$$ X = v\, p_x\, q_x\, \ddot{a}_{x+1:x+1} $$

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