Quantitative Economics / Faculty of Economics and Business
University of Amsterdam

Exam Dynamical Systems

Date: Wednesday June 25, 2008 Time: 9:00-12:00

- Write down your name and student ID on everything you hand in.
- This is *not* an open book exam.
- Motivate your answers. No motivation = no points.
- This exam consists of four questions.

Question 1:
Consider the dynamical system $x_{t+1} = Ax_t$, where

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix},$$

and $x(0) = (1, 1)$ is given. Give the solution of this system.

Question 2:
Consider the following dynamical system:

$$\begin{aligned} \dot{x} &= \frac{1}{8}x^2 - y, \\ \dot{y} &= y^2 - x. \end{aligned}$$

a) Determine the equilibria of this system and their (asymptotic) stability if possible.

Suppose we extend the dynamical system in the following way:

$$\begin{aligned} \dot{x} &= \frac{1}{8}x^2 - y, \\ \dot{y} &= y^2 - x, \\ \dot{z} &= -z + G(x, y), \end{aligned}$$

where $G : \mathbb{R}^2 \to \mathbb{R}$ a differentiable function.

b) Find the equilibria of this system (in terms of G) and determine their stability.

Question 3:
The flow of a dynamical system is given by:

$$\Phi(t, x) = \frac{x}{x + (1 - x)e^t}.$$

The corresponding differential equation will be denoted by $\dot{x} = f(x)$.

a) Give the definition of a flow.

b) Show that $\Phi(t, x)$ is indeed a flow.

c) Find the function $f(x)$.

Question 4:
Are the following propositions true or false? Explain why.

a) Let $x_{n+1} = x_n + x_{n-1}$ and $x_0 = x_1 = 1$. Define $r_n := x_{n+1}/x_n$. Then $\lim_{n\to\infty} r_n = \frac{1}{2} + \frac{1}{2}\sqrt{5}$.

b) Let $f(x) = \sqrt[3]{x} + x^2 + e^x$. The dynamical system given by $\dot{x} = f(x)$ has a unique solution for any initial condition.

c) Let $F : \mathbb{R}^2 \to \mathbb{R}$ be a strictly concave function and let ∇F denote the gradient of F. A fixed point of the dynamical system $\dot{x} = \nabla F(x)$ is asymptotically stable.