Uitwerkingen tentamen OR1S, 07-07-2000

1 Toestanden $\{0, 1, 2, \ldots\}$

Stroomdiagram:

$\lambda = 0.15$, $\mu = 1/5 = 0.2$

Balansvergelijkingen:

a1: Per toestand $\{k\}$

$$\begin{cases} p_k(\lambda+\mu) = p_{k-1}\lambda + p_{k+1}\mu & , k \geq 1 \\ p_0\lambda = p_1\mu \end{cases}$$

a2: Per verzameling $\{0, 1, \ldots, k\}$

$$p_k\lambda = p_{k+1}\mu \quad , k \geq 0$$

b) Oplossen balansvergelijkingen geeft

$$p_k = \left(\frac{\lambda}{\mu}\right)^k p_0 = \varrho^k(1-\varrho) \quad , \varrho = \frac{\lambda}{\mu} = \frac{0.15}{0.2} = 3/4$$

$$L = \sum_{k=0}^{\infty} k p_k = \frac{\varrho}{1-\varrho} = \frac{3/4}{1-3/4} = 3$$

$$L = \lambda S \text{ (Little)} \Rightarrow S = L/\lambda = 20$$

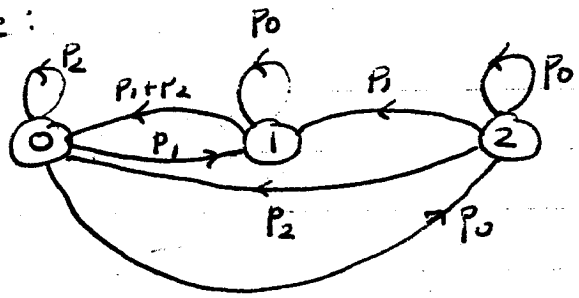
2. Neem als toestand voorraad aan het eind van de dag, dus 0, 1, of 2

Overgangsmatrix

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} p_2 & p_1 & p_0 \\ p_1+p_2 & p_0 & 0 \\ p_2 & p_1 & p_0 \end{pmatrix} \end{array}$$

$p_0 = 0.6$, $p_1 = 0.3$, $p_2 = 0.1$

Plaatje:



Evenwichtsverdeling π

$$\begin{cases} \pi_0 = \pi_0 p_2 + \pi_1 (p_1 + p_2) + \pi_2 p_2 \\ \pi_1 = \pi_0 p_1 + \pi_1 p_0 + \pi_2 p_1 \\ \pi_2 = \pi_0 p_0 + \pi_2 p_0 \end{cases}, \quad \pi_0 + \pi_1 + \pi_2 = 1$$

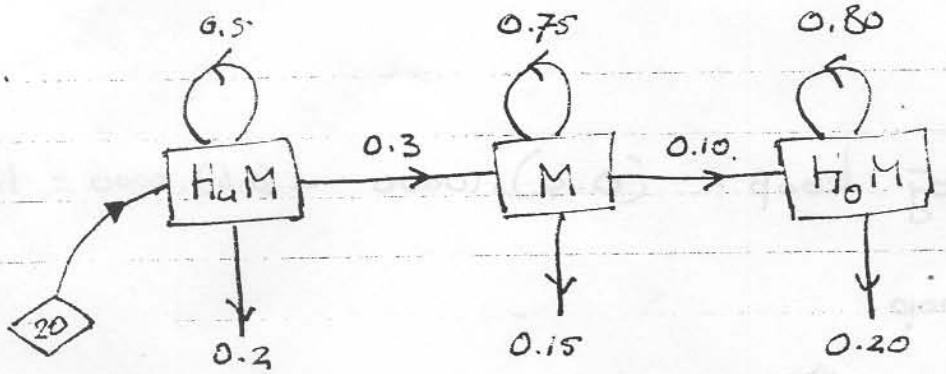
Oplossing:

$$(\pi_0, \pi_1, \pi_2) = \frac{1}{35}(8, 15, 12)$$

Per dag $\frac{8}{35}$ bestelling en $\frac{15}{35} p_2$ tekort

Kosten $\frac{8}{35} \cdot 100 + \frac{15}{35} \cdot \frac{1}{10} \cdot 200 = \frac{1100}{35} \approx 31.4$ gulden/dag

3.



$N(2000) = (30, 40, 30)$

$N(2001) = (20 + (0.5)\cdot 30,\ (0.3)\cdot 30 + (0.75)\cdot 40,\ (0.1)\cdot 40 + (0.8)\cdot 30)$

$= (35, 39, 28)$

$N(2002) = (37.5, 39.75, 26.3)$

Op een duur:

$N_{H_4M}(\infty) = 20 + 0.5\, N_{H_4M}(\infty) \Rightarrow N_{H_4M}(\infty) = 40$

$N_M(\infty) = 0.3\, N_{H_4M}(\infty) + 0.75\, N_M(\infty) \Rightarrow N_M(\infty) = 48$

$N_{H_0M}(\infty) = 0.1\, N_M(\infty) + 0.8\, N_{H_0M}(\infty),\ N_{H_0M}(\infty) = 24$

Notatie: $N = (\#H_4M, \#M, \#H_0M)$

4.

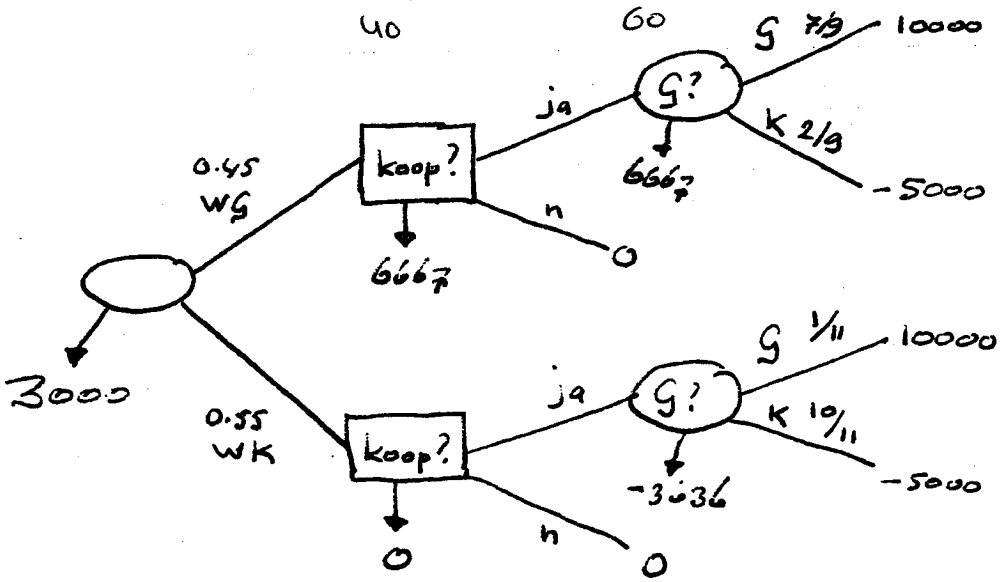
a) Profyt bij koop: $(0.4).10000 - (0.6).5000 = 1000$

Dus koop

100

b)

	G	K	
WG	0.35 (40 · 7/8)	0.10 (60 · 1/6)	0.45
WK	0.05 (40 · 1/8)	0.50 (60 · 5/6)	0.55
	0.4	0.6	
	40	60	



Bij advies WG is de kans op G $\frac{35}{45} = 7/9$
Dan levert 'Koop' $7/9 * 10000 - 2/9 * 5000 = 6667,=$ op.
Bij advies WK is de kans op G maar 1/11.
kopen is een onverstandig (verwachte opbrengst is -3636).
Verwachte opbrengst 'met' kennis : $(.45) * 6667 = 3000,=$