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SOME TEST QUESTIONS

Note: These questions are purely designed to help you to assimilate and understand the material covered in the classes. Similar questions may or may not be a part of the final exam.

1. The variable $y_t = \Delta \ln(P_t)$ represents the daily returns on wheat, where P_t is the opening daily cash price of wheat on the Chicago Merchandise Commodity Exchange. The following model has been proposed to explain the behavior of the wheat price series:

$$\begin{aligned} y_t &= z_t\sigma_t \\ \sigma_t^2 &= \omega + \alpha_t y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \beta_1\sigma_{t-1}^2 \\ z_t &\sim i.i.d.(0,1) \end{aligned}$$

   (a) Express the above GARCH(1,2) process as an infinite order ARCH process.
   (b) Write down the non-negativity conditions for the process.
   (c) What is the unconditional variance and autocorrelation function of the returns series y_t?
   (d) Under what conditions will the y_t process be covariance stationary?
   (e) Find the ARMA(p,q) representation of y_t^2. Note: You need to find the orders p and q, and the ARMA parameters in terms of the parameters of the original model.
   (f) Without doing any detailed calculations, describe the properties of the theoretical population autocorrelation function of y_t^2.

2. Consider the autoregressive, AR(2) time series process given by:

$$y_t - y_{t-1} + (2/9)y_{t-2} = \varepsilon_t \qquad (1)$$

   where ε_t is a white noise process.

   (a) Show that the process is stationary.
   (b) The Wold decomposition of the above process is given by $y_t = \sum_{k=0}^{\infty}\Psi_k\varepsilon_{t-k}$. Show that the expression for the typical coefficient is given by, $\Psi_k = 2(\frac{2}{3})^k - (\frac{1}{3})^k$ for $k \geqslant 0$, and find the numerical values of Ψ_k for $k = 0, 1, 2, 3$.
   (c) Write down the first two Yule Walker equations in terms of the population autocorrelations, ρ_k, for the process in equation (1). Show that $\rho_1 = (9/11)$, and hence show that the typical autocorrelation coefficient at lag k is given by

$$\rho_k = (\frac{16}{11})(\frac{2}{3})^k - (\frac{5}{11})(\frac{1}{3})^k, \quad \text{for} \quad k \geqslant 0.$$

   (d) Is the following ARMA(2,1) model $y_t - y_{t-1} + (2/9)y_{t-2} = \varepsilon_t - (1/3)\varepsilon_{t-1}$ a sensible model to use? Justify your answer.

3. Suppose an economic time series is considered to possibly be non stationary. The following model has been proposed,

$$\begin{aligned} y_t &= \alpha + \beta t + u_t \\ u_t &= \varepsilon_t - \theta\varepsilon_{t-1} \end{aligned}$$

   where $-1 < \theta < 1$, ε_t is a white noise process.

   (a) Does the autocorrelation function for y_t exist? If so, write down the autocorrelation function.
   (b) Explain the distinction between trend and difference stationary processes and why the distinction is important in time series econometrics.

4. Consider the following univariate AR(2) process:

$$y_t = \varphi y_{t-1} + (1-\varphi)y_{t-2} + \varepsilon_t \qquad (2)$$

   where ε_t is white noise.

   (a) Show that the above process has at least one root equal to 1 if an only if $0 < \varphi \leqslant 2$.
   (b) Explain why the error correction form (ECM) of (2) becomes

$$\Delta y_t = (\varphi - 1)\Delta y_{t-1} + \varepsilon_t$$

   For which φ in the interval $< 0, 2]$ is $y_t \sim I(1)$, and for which φ is $y_t \sim I(2)$?

5. Let $X_{1t} = \sum_{i=1}^{t}\varepsilon_{1i}$ and $X_{2t} = \varepsilon_{2t}$, where $\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$ are two independent sequences of i.i.d. random variables.

   (a) Show that X_{2t} is I(0).
   (b) Show that X_{1t} is I(1).
   (c) Show that $X_t = X_{1t} + X_{2t}$ is I(1).

6. Consider the following bivariate model

$$\begin{aligned} y_t - y_{t-1} &= \gamma(y_{t-1} - x_{t-1}) + \alpha(y_{t-1} - y_{t-2}) + v_t \\ x_t - x_{t-1} &= \beta(y_{t-1} - y_{t-2}) + w_t \end{aligned}$$

   where v_t and w_t are mutually independent white noise variables with variances σ_v^2 and σ_w^2 and covariance σ_{vw}.

   a. Show that there is cointegration between y_t and x_t.
   b. Write the two equations as a VAR model for the variables y_t and x_t.