Example Exam Advanced Econometrics 1 (FEE-UvA)
00 November, 20??; 14.00-16.00 hrs.

This is NOT an "open-book" exam; apart from pen and paper no further aids and tools are allowed to be used. Write your name and student number on all sheets that you hand in for marking. For each separate (sub-)question the maximum score/weight is mentioned between brackets. The sum of these weights is 110 in total. However, only your best-answered questions, together giving a maximum score of 75, will be taken into account. **Hence, arbitrary sub-questions, together worth 35 points, should be skipped.** The final grade (scaled 1 through 10; $fail < 6.0 \leq pass$) will be determined for 75% by this written exam and for 25% by the three theory and three computer assignments (if these were handed in before their respective deadlines).

The grades will become available within 3 weeks and will be announced by the FEE student administration office. Individual participants may inspect their results by making an appointment (by email) with the responsible professor.

Note that your handwriting should be clear, your notation consistent, and all your answers should be motivated. Below, we adopt the notation as used in Davidson & MacKinnon (2004), but we do not use bold-face for vectors and matrices, and use $'$ for transpose.

1. State the essentials (without proof) of the two theorems indicated below. Use not more than about 10 lines per theorem, clarify your notation and clearly state the assumptions required for the theorem to hold.
   - (a) {10} Frisch-Waugh-Lovell Theorem.
   - (b) {10} Central Limit Theorem (simplest form).

2. Consider the general nonlinear regression model $y_t = x_t(\beta) + u_t$, with $u_t \mid x_t(\beta) \sim \text{IID}(0, \sigma^2)$, $t = 1, ..., n$. The coefficient vector β contains k elements. The NLS criterion function $Q(\beta) = \frac{1}{n}\sum_{t=1}^{n}(y_t - x_t(\beta))^2 = \frac{1}{n}(y - x(\beta))'(y - x(\beta))$ has gradient vector $g(\beta) = -\frac{2}{n}X(\beta)'(y - x(\beta)) = -\frac{2}{n}\sum_{t=1}^{n}(y_t - x_t(\beta))X_t(\beta)'$, where $X(\beta) = \frac{\partial}{\partial \beta'}x(\beta)$ has t^{th} row $X_t(\beta)$. Newton's method to find the minimum of $Q(\beta)$ involves the iteration scheme $\beta_{(l+1)} = \beta_{(l)} - [H(\beta_{(l)})]^{-1}g(\beta_{(l)})$, for $l = 0, 1, 2, ...$, with $\beta_{(0)}$ an initial estimate of β. Suppose that the DGP has true parameters β_0 and σ_0^2.
   - (a) {10} Derive the typical element $H_{ij}(\beta)$ of the Hessian matrix $H(\beta)$ of $Q(\beta)$ for $i, j = 1, ..., k$.
   - (b) {10} Explain why $H(\beta_0)$ is asymptotically equivalent to $\frac{2}{n}X(\beta_0)'X(\beta_0)$.
   - (c) {5} Show how the above yields the Gauss-Newton method.
   - (d) {5} Indicate how the R^2 of the Gauss-Newton regressions can be used to establish the convergence of the NLS procedure.

3. Consider the specific nonlinear regression model $y_t = X_t\beta + \gamma(X_t\beta)^2 + u_t$, with $u_t \mid X_t \sim \text{IID}(0, \sigma^2)$, $t = 1, ..., n$. Again, β is a $k \times 1$ vector, and so is X_t', but note that these vectors X_t and β are related to, but are different from $X_t(\beta)$ and β in Question 2, where they relate to the full nonlinear regression function.

(a) {5} Discuss any specific requirements concerning the data X_t and coefficients β and γ for avoiding identification related problems when calculating the nonlinear least squares (NLS) estimator.

(b) {5} Why would you prefer the NLS estimator and not a particular appropriate alternative consistent method of moments estimator?

(c) {5} Describe the series of Gauss-Newton regressions that will yield the NLS estimator for this particular model.

(d) {5} Consider the 1-step Gauss-Newton regression that starts from the restricted estimator which imposes $\gamma = 0$ and indicate its particulars (specify regressand and regressors).

(e) {5} Indicate how the above can be used to test $H_0 : \gamma = 0$. What is the test statistic?

(f) {5} Prove that an algebraically equivalent test is obtained from the auxiliary regression $y_t = X_t\beta + \gamma\tilde{y}_t^2 + v_t$, where $\tilde{y}_t = X_t\tilde{\beta}$, with $\tilde{\beta}$ the OLS estimator in the regression of y_t on X_t (this is known as a RESET test).

4. Consider the classical normal linear regression model $y = X_1\beta_1 + X_2\beta_2 + u$, where β_1 has k_1 and β_2 has k_2 components, $k_1 + k_2 = k$. For the DGP $u \mid X \sim \text{N}(0, \sigma_0^2 I_n)$. We examine testing $H_0 : \beta_2 = \beta_{20}$ by the Wald test statistic, which has generic form $(\hat{\theta} - \theta_0)'[\widehat{\text{Var}}(\hat{\theta})]^{-1}(\hat{\theta} - \theta_0)$. Note that the unrestricted estimator of β_2 is $\hat{\beta}_2 = (X_2'M_1X_2)^{-1}X_2'M_1y$, with $M_1 = I_n - X_1(X_1'X_1)^{-1}X_1'$.

(a) {5} Derive the variance of $\hat{\beta}_2$.

(b) {5} Show that for $\hat{\sigma}^2 = \frac{1}{n}\hat{u}'\hat{u}$, where $\hat{u}$ are the least squares residuals of the unrestricted model, one finds $E(\hat{\sigma}^2) \leq \sigma_0^2$.

(c) {5} Show that the Wald statistic, under H_0, can be expressed as $\frac{u'M_1X_2(X_2'M_1X_2)^{-1}X_2'M_1u}{\hat{\sigma}^2}$.

(d) {5} State a particular theorem that enables you to obtain the distribution of $\frac{n}{\sigma_0^2}\hat{\sigma}^2$ and of $\frac{1}{\sigma_0^2}u'M_1X_2(X_2'M_1X_2)^{-1}X_2'M_1u$, respectively. What are these distributions?

(e) {5} Demonstrate that the components $\hat{\sigma}^2$ and $u'M_1X_2(X_2'M_1X_2)^{-1}X_2'M_1u$ are independent.

(f) {5} Express exact critical values for this Wald test statistic in those of the F distribution.

Success!