

Exam Advanced Econometrics 1 (FEE-UvA: 2004/2005)
resit of January 27, 2005; 9.30-11.30 hrs.

This is NOT an “open-book” exam; apart from pen and paper no further aids and tools are allowed to be used. Write your name and student number on all sheets that you hand in for marking. For each separate (sub-)question the maximum score/weight is mentioned between brackets. The sum of these weights is 105 in total. However, only your best-answered questions, together giving a maximum score of 75, will be taken into account. **Hence, arbitrary sub-questions, together worth 30 points, should be skipped.** The final grade (scaled 1 through 10; *fail* < 6.0 ≤ *pass*) will be determined for 75% by this written exam and for 25% by the three theory and three computer assignments (if these were handed in before their respective deadlines).

The grades will become available within 3 weeks and will be announced by the FEE student administration office. Individual participants may inspect their results by making an appointment (preferably by email) with the responsible professor.

Note that your handwriting should be clear, your notation consistent, and all your answers should be motivated. Below, we adopt the notation as used in Davidson & MacKinnon (2004), but we do not use bold-face for vectors and matrices, and use ' for transpose.

1. State the essentials (without proof) of the two theorems indicated below. Use not more than about 10 lines per theorem, clarify your notation and clearly state all the assumptions required for the theorem to hold.
 - (a) {10} Gauss-Markov Theorem.
 - (b) {10} Law of Large Numbers (simplest form).

2. Consider the case of underspecification in the classical linear regression model, where the DGP is $y = X\beta_0 + Z\gamma_0 + u$ with $u \sim IID(0, \sigma_0^2 I)$ and X and Z exogenous, whereas the model actually estimated by OLS is $y = X\beta + u$.
 - (a) {5} Derive the expectation of this OLS estimator $\hat{\beta}$.
 - (b) {5} Examine and describe the special circumstances under which $\hat{\beta}$ is (un)biased, and any effects on this by the size of the sample.
 - (c) {5} Derive the mean squared error matrix $MSE(\hat{\beta}) \equiv E[(\hat{\beta} - \beta_0)(\hat{\beta} - \beta_0)']$.
 - (d) {5} Let $\tilde{\beta}$ be the OLS estimator in the appropriately specified model. Derive $MSE(\tilde{\beta})$.
 - (e) {5} What conclusions can you draw regarding differences in efficiency between $\hat{\beta}$ and $\tilde{\beta}$?
 - (f) {10} Consider testing $\gamma = 0$ in the adequately specified model $y = X\beta + Z\gamma + u$ by the F-test. When X contains k variables and Z has r columns the test statistic can be expressed as

$$\frac{n - k - r}{r} \frac{y' M_X Z (Z' M_X Z)^{-1} Z' M_X y}{y' M_{XZ} y},$$

which can be rewritten as

$$\frac{n - k - r}{n} \frac{\frac{1}{r\sigma_0^2} y' M_X Z (Z' M_X Z)^{-1} Z' M_X y}{\frac{1}{n\sigma_0^2} y' M_{XZ} y},$$

where for $n \rightarrow \infty$ we have $\frac{n-k-r}{n} \rightarrow 1$ and $\frac{1}{n\sigma_0^2} y' M_X Z y \rightarrow 1$, so that the test statistic is asymptotically equivalent with

$$\frac{1}{r\sigma_0^2} y' M_X Z (Z' M_X Z)^{-1} Z' M_X y.$$

Obtain the limiting distribution of the test statistic under $\gamma = 0$, upon referring explicitly to any theorems that you use while indicating why they are applicable.

- (g) {5} Now consider the limiting behavior of the test statistic under $\gamma \neq 0$, and address its power.

3. Consider the model

$$y_t = X_t \beta + u_t \quad (1)$$

with $E(u_t | \Omega_t) = 0$ and $E(u_t^2 | \Omega_t) = h(\delta + Z_t \gamma) > 0$, where the $1 \times k$ vector X_t and the $1 \times r$ vector Z_t of observed variables both belong to the information set Ω_t , $t = 1, \dots, n$. The matching vectors β and γ and the scalar δ are unknown parameters, and also the function $h(\cdot) > 0$ is unknown. Let $v_t \equiv u_t^2 - E(u_t^2 | \Omega_t)$ then we have the non-linear regression model

$$u_t^2 = h(\delta + Z_t \gamma) + v_t, \quad (2)$$

which, in case the u_t^2 had been observed, could be estimated by exploiting the Gauss-Newton regression (with obvious notation)

$$u_t^2 - h(\delta + Z_t \gamma) = h'(\delta + Z_t \gamma) b_\delta + h'(\delta + Z_t \gamma) Z_t b_\gamma + error. \quad (3)$$

We are interested in estimating β and in finding an operational test for $\gamma = 0$.

- (a) {10} How would you estimate β when $h(\delta + Z_t \gamma) = h_t$ were known? Give an expression for the estimator and derive its most important properties (regarding its first two moments and its efficiency).
- (b) {5} Consider OLS estimation of β in (1) by, say, $\tilde{\beta}$. What are the most important properties of $\tilde{\beta}$?
- (c) {10} If u_t^2 were observed, how could you employ $\tilde{\beta}$ and (3) to obtain 1-step estimators of γ , δ and β ? What are the most important properties of these 1-step estimators (no derivation/proof required)?
- (d) {10} Use the above to clarify that $\gamma = 0$ can be tested, without knowing $h(\cdot)$ nor observing u_t^2 , simply by performing an F-test on the significance of the regressors Z_t in the artificial regression of the squared OLS residuals \tilde{u}_t^2 on an intercept and Z_t .
- (e) {10} Clarify why an asymptotically equivalent test can be obtained on the basis of the R^2 of the same artificial regression and indicate what the test statistic is, and what critical value has to be used.

Success!