Exam Advanced Econometrics 1 (FEE-UvA: 2004/2005)
resit of June 28, 2005; 10.00-12.00 hrs.

This is NOT an "open-book" exam; apart from pen and paper no further aids and tools are allowed to be used. Write your name and student number on all sheets that you hand in for marking. For each separate (sub-)question the maximum score/weight is mentioned between brackets. The sum of these weights is 105 in total. However, only your best-answered questions, together giving a maximum score of 75, will be taken into account. **Hence, arbitrary sub-questions, together worth 30 points, should be skipped.** The final grade (scaled 1 through 10; $fail < 6.0 \leq pass$) will be determined for 75% by this written exam and for 25% by the three theory and three computer assignments (if these were handed in before their respective deadlines).

The grades will become available within 3 weeks and will be announced by the FEE student administration office. Individual participants may inspect their results by making an appointment (preferably by email) with the responsible professor.

Note that your handwriting should be clear, your notation consistent, and all your answers should be motivated. Below, we adopt the notation as used in Davidson & MacKinnon (2004), but we do not use bold-face for vectors and matrices, and use $'$ for transpose.

1. Let $X = (X_1\ X_2)$ be a $n \times k$ matrix of full column rank, whereas X_1 has k_1 columns, with $0 < k_1 < k$. The matrix A is $k \times k$ and nonsingular. By P_Z we denote the matrix that projects on the space spanned by the columns of the matrix Z, whereas $M_Z = I - P_Z$. Indicate clearly what properties/rules you use when proving/deriving:

   (a) {5} $P_A = P_{A^{-1}} = P_I = I$.

   (b) {5} $P_{XA} = P_X$.

   (c) {5} $P_{X_1} P_X = P_{X_1}$.

   (d) {5} Derive $\text{Tr}(M_{X_2})$.

   (e) {15} Prove $P_{M_{X_2} X_1} = P_X - P_{X_2}$.

2. Consider the DGP $y = X\beta_0 + u$, where X is $n \times k$ and $u \sim \text{IID}(0, \sigma_0^2)$. The regressors X are exogenous; β_0, σ_0^2 and the actual distribution of u are unknown.

   (a) {5} What is meant by the exogeneity of X and what important consequences does it have for OLS estimation?

   (b) {5} Focus on estimating the model $y = X_1\beta_1 + x_2\beta_2 + u$, where $X = (X_1\ x_2)$ and X_1 has $k-1$ columns. Give the expression for the OLS estimator of β_2 and derive its variance.

   (c) {5} For testing $\beta_2 = 0$, what test statistic would you use? Give the expression for the test statistic in all detail.

   (d) {10} Under $\beta_2 = 0$ and $u \sim \text{IID}(0, \sigma_0^2)$ what can you say about the distribution of that test statistic? Indicate what arguments/theorems you use in order to obtain this distribution.

   (e) {5} Indicate how you could easily adapt the above derivations to cover the situation where you want to test $\beta_2 = \beta_{20}$, with β_{20} an arbitrary real number.

(f) {10} Sketch the essentials of the power function of the above test statistic, in particular any effects of n and the true value of β_0 (to keep things relatively simple you may assume now that u is Normal).

3. Consider the model $y_t = \beta_1 + \beta_2 z_t^{\beta_3} + u_t$, with $u_t \sim \text{IID}(0, \sigma^2)$.

   (a) {5} What criterion function has to be minimized in order to obtain the NLS (nonlinear least squares) estimator $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)'$ and what moment conditions will be satisfied by these NLS estimates?

   (b) {5} For which particular situations will identification problems emerge?

   (c) {5} Present the particular artificial regression from which it is easy, provided $\hat{\beta}$ has been found, to obtain an estimate $\widehat{\text{Var}}(\hat{\beta})$ of $\text{Var}(\hat{\beta})$. Give an appropriate expression for $\widehat{\text{Var}}(\hat{\beta})$.

   (d) {5} Due to introduction of the Euro it seems desirable to transform the data for y_t and z_t, giving $y_t^* = y_t/c$ and $z_t^* = z_t/c$ with appropriate c, and then estimate model $y_t^* = \beta_1^* + \beta_2^* z_t^{*\beta_3^*} + u_t^*$, with $u_t^* \sim \text{IID}(0, \sigma^{*2})$. Make clear how the new parameters are related to the former ones.

   (e) {10} Use the delta-method to indicate how $\hat{\beta}^*$ and $\widehat{\text{Var}}(\hat{\beta}^*)$ can be obtained directly from $\hat{\beta}$ and $\widehat{\text{Var}}(\hat{\beta})$.

Success!