

Example Exam A Advanced Econometrics 2 FEB-UvA
00 January, 200?; 14.00-16.00 hrs.

This is NOT an “open-book” exam; apart from pen and paper no further aids and tools are allowed to be used. Write your name and student number on all sheets that you hand in for marking. For each separate (sub-)question the maximum score/weight is mentioned between brackets. The sum of these weights is 105 in total. However, only your best-answered questions, together giving a maximum score of 75, will be taken into account. **Hence, arbitrary sub-questions, together worth 30 points, should be skipped.** The final grade (scaled 1 through 10; $fail < 6.0 \leq pass$) will be determined for 75% by this written exam and for 25% by the three theory and three computer assignments (if these were handed in before their respective deadlines).

The grades will become available within 3 weeks and will be announced by the FEE student administration office. Individual participants may inspect their results by making an appointment (preferably by E-mail) with the responsible professor.

Note that your handwriting should be clear, your notation consistent, and all your answers should be motivated. Below, we adopt the notation as used in Davidson & MacKinnon (2004), but we do not use bold-face for vectors and matrices, and use $'$ for transpose.

1. Consider the non-linear regression model $y_t = x_t(\beta) + u_t = \beta_1 + \frac{\beta_2}{z_t - \beta_3} + u_t$, where the z_t are non-random and the u_t are IID with density $f(u_t) = (2\pi\sigma^2)^{-1/2} \exp(-u_t^2/2\sigma^2)$, $t = 1, \dots, n$.
 - (a) {5} Give the loglikelihood for the full sample y_1, \dots, y_n .
 - (b) {5} Derive expressions for the 4 elements of the score vector.
 - (c) {5} Derive the essentials of the Hessian matrix.
 - (d) {10} Provide an expression for the estimator based on the information matrix for the variance of the ML estimator for $\beta = (\beta_1, \beta_2, \beta_3)'$.
 - (e) {10} How would you test $\beta_3 = \beta_3^0$ by a Wald test? Give the test statistic, the rejection region and the major qualifications of the test.

2. Let $\hat{\beta}^{(1)}, \dots, \hat{\beta}^{(R)}$ be a sample of independent and correctly programmed drawings from the distribution of an estimator $\hat{\beta}$ for parameter β . Let β be scalar. It may be the case that $\hat{\beta}$, which has CDF F , does not have finite moments. For any real x the EDF (empirical distribution function) of $\hat{\beta}$ is defined as $\hat{F}(x) \equiv \frac{1}{R} \sum_{r=1}^R I(\hat{\beta}^{(r)} \leq x)$, where $I(\cdot)$ is the 0 – 1 indicator function.
 - (a) {10} Consider the random variables $v_r \equiv I(\hat{\beta}^{(r)} \leq x)$, $r = 1, \dots, R$. Characterize the distribution of these v_r (what kind of distribution, give first two moments).
 - (b) {10} Present and clarify the characteristics (probability limit, asymptotic distribution function and its moments) of $\hat{F}(x)$ for $R \rightarrow \infty$.
 - (c) {5} Indicate whether and why the (asymptotic) distribution of $\hat{F}(x)$ is (not) affected by the (non-)existence of the moments of $\hat{\beta}$.
 - (d) {10} Discuss the accuracy of $\hat{F}(x)$ with respect to estimating $F(x)$. If one has the intention to restrict $|\hat{F}(x) - F(x)|$ to 10^{-3} , irrespective of the value of x , how large would you choose R ? Why?

3. Consider the system of g linear simultaneous equations ($i = 1, \dots, g$)

$$y_i = X_i \beta_i + u_i, \quad (1)$$

where X_i is $n \times k_i$ may contain some of the y_j for $j \neq i$. The full system can be written

$$y_{\bullet} = X_{\bullet} \beta_{\bullet} + u_{\bullet}, \quad (2)$$

where $y_{\bullet} = (y'_1, \dots, y'_g)'$ and $u_{\bullet} = (u'_1, \dots, u'_g)'$ are $ng \times 1$ vectors, with $E(u_{\bullet}) = 0$ and $E(u_{\bullet} u'_{\bullet}) = \Sigma \otimes I_n$. Suppose that W collects all the exogenous and predetermined variables contained in X_1, \dots, X_g and has full column rank l . Then we can solve for all the endogenous variables and obtain the SUR (seemingly unrelated regression) system

$$y_i = W \pi_i + v_i, \quad (3)$$

with $E(v_{it} | W_t) = 0$, for $t = 1, \dots, n$ and W_t the t -th row of W . This SUR system can be written as

$$y_{\bullet} = (I_g \otimes W) \pi_{\bullet} + v_{\bullet}, \quad (4)$$

where $E(v_{\bullet} v'_{\bullet}) = \Sigma^* \otimes I_n$.

- (a) {5} Give the proper definitions (contents and dimensions) of X_{\bullet} , β_{\bullet} and π_{\bullet} .
- (b) {10} System (4) is a very particular SUR system. Prove that the single equation OLS estimators of (3) are efficient, irrespective of the nature of Σ^* .
- (c) {10} Indicate why we will obtain the efficient GMM estimator of β_{\bullet} by solving $\hat{X}'_{\bullet} (\Sigma^{-1} \otimes I_n) (y_{\bullet} - X_{\bullet} \hat{\beta}_{\bullet}) = 0$, where $\hat{X}_{\bullet} = (I_g \otimes P_W) X_{\bullet}$.
- (d) {10} Specify a special situation under which the estimator $\hat{\beta}_{\bullet}$ consists of the single equation IV (or 2SLS) estimators of (1), and prove that this is indeed the case.

Success!