



## Faculty of Economics and Business

**On this first page, you will find important information on the examination.**

**Before starting with the examination you should read this information!**

**Examination:** Market-Consistent Pricing and Embedded Value 80014

**Date and time of the examination:** Tue 28 Oct 2008 09:00 - 12:00

**Duration examination:** 3 hours

You have to identify yourself with the UvA-identification card or your UvA studentcard and passport or drivers' license or any other valid proof of identification for students containing a photograph.

If you did not register for this examination, your result will not be recorded. To record your result officially, please go to the Student Desk. After paying a fee for administration costs your result will be recorded.

**Add your name and studentnumber to every  
sheet of paper you hand in.**

**Warning against fraud:** Do not commit fraud! In the case of fraud the maximum punishment is exclusion to all examinations for one year.

Your mobile phone should be switched off and should be put in your briefcase. Your briefcase should be closed and standing on the floor to the left of your desk.

During the examination you are not allowed to go to the toilet unless the coordinating invigilator gives you permission to do so.

**Tools allowed:** pen, (non-graphical) calculator

**Specific information on this examination:** This exam consists of 1 page. You have 3 questions, each earning the indicated number points, giving a maximum number of 14 points. The result of this exam (75%) is averaged with the grade of your homework assignments (25%). Final grade will be rounded to an integer number.

**The result of this examination will be published within 18 working days after the date of this examination. In the case that the re-examination is programmed within 6 weeks of this examination, the results will be published within 12 days.**

**Reviewing the examination:** Make an appointment with Pelsser ([a.a.j.pelsser@uva.nl](mailto:a.a.j.pelsser@uva.nl))

A copy of this examination and a copy of the solutions will be published on BlackBoard.

**Good luck!**

### Assignment 1: Discounting Cash Flows (5 points)

Consider the following set of par swap-rates:

| Maturity | Swap-rate |
|----------|-----------|
| 1 year   | 4.00%     |
| 2 year   | 4.25%     |
| 3 year   | 4.50%     |
| 4 year   | 4.75%     |
| 5 year   | 5.00%     |

- Determine the discount factors implied by these swap rates.
- Represent the discount factors as zero-rates using *annual compounding*.
- Calculate the par swap-rate of a forward starting swap that starts at  $t=2$  and expires at  $t=5$ .
- Consider a contract with a maturity of 5 years. It has cash flows that linearly decline from 1000 at  $t=0$  to 600 in  $t=5$ . Calculate the market-consistent value of this contract.
- Discuss the following statement: "Discounting cash flows with the term-structure of interest rates is an arbitrary choice, just as arbitrary as discounting at a fixed interest rate of 4%".

### Assignment 2: Utility Indifference Pricing (4 points)

Consider an economic agent with a utility function  $U(x) = \exp(-ax)$ . Consider an insurance portfolio  $P$  which has a probability distribution of claims at time  $T$  equal to a normal distribution with mean  $\mu_P$  and standard deviation  $\sigma_P$ . Suppose the terminal wealth at time  $T$  is given by a certain (i.e. non-random) amount  $W$ .

- Calculate the utility indifference price  $\pi_P$  by solving the equation  $U(W) = \mathbf{E}[U(W + \pi_P - P)]$ .  
**Hint:** the moment generating function of a random variable  $X$  with a normal distribution  $n(\mu, \sigma^2)$  is given by  $\mathbf{E}[\exp(tX)] = \exp(t\mu + \frac{1}{2}t^2\sigma^2)$  for all  $t$ .
- Explain how you can decompose the indifference price  $\pi_P$  into a "Best Estimate" and a "MVM" component.

Consider now an additional insurance claim  $I$  with a normal distribution mean  $\mu_I$  and standard deviation  $\sigma_I$ . The correlation between the claim  $I$  and the existing portfolio  $P$  is given by the parameter  $\rho$  (with  $-1 \leq \rho \leq 1$ ).

- Calculate the utility indifference price  $\pi_{P+I}$  of the combined portfolio  $P+I$ .
- Consider the "marginal" indifference price  $\pi_I$  of the claim  $I$  given by  $\pi_I = \pi_{P+I} - \pi_P$ . Discuss the properties of the price  $\pi_I$  for  $\rho < 0$ ,  $\rho = 0$  and  $\rho > 0$ .

### Assignment 3: Unit-Linked Contract (4 points)

Consider a binomial tree economy with parameters  $R=1.05$  (i.e. interest rate = 5%) and  $u=1.25$  and  $d=0.8$  and with 4 time-steps of 1 year. Hence, after 1 year a stock-index with initial value  $S$  can either increase to the value  $uS$  with probability  $p=0.5$ , or decrease to  $dS$  with probability  $(1-p)=0.5$ ; a bond with initial value  $B$  increases after 1 year to  $RB$  with certainty.

- Construct the replicating portfolio for a "one-step" binomial tree for a derivative that pays out  $f_u$  in the "u-state" and  $f_d$  in the "d-state".

Consider a unit-linked insurance contract that invests initially 100 in the stock-index  $S$ . At year 4, which is the maturity date of this contract, the policyholder has the right to choose between the value of the stock index, or the initial investment of 100.

- Construct the 4-step binomial tree. Determine for every node the replicating portfolio and the market-value of the replicating portfolio.
- What is today's market-consistent value of this unit-linked contract?
- Discuss any pattern that you see in the replicating portfolios for the 4-step tree.