# Final Exam Financial Econometrics, 05-07-2005

The exam lasts for 3 hours. This is not an open-book exam. The use of pocket calculators is allowed.

For each question the maximum number of points that can be obtained is indicated, adding up to 110 points. Only the best-answered questions, together worth 100 points, will be taken into account; therefore, some questions, together worth 10 points, may be skipped.

Write down your name and student number on all your work. Always give motivated answers; correct answers with no motivation will not be awarded the full amount of points.

The grades for this exam will be made available on or before Tuesday, July 26, 2005.

1. **[10]** Derive the autocovariances and the autocorrelation function for an MA($q$) process.

2. Consider the covariance stationary ARCH(1) process $u_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$ and $\varepsilon_t \sim$ IIN(0,1).

   (a) **[5]** What are the first four moments of $u_t$, conditional on $u_{t-1}, u_{t-2}, \ldots$?

   (b) **[10]** Derive the first four unconditional moments of $u_t$, stating conditions under which they exist.

3. **[10]** Indicate what are the essentials of "spurious regression": what is it, when does it occur, are there any precautions against it?

4. Consider the dynamic Gordon growth model

$$p_t = \frac{k}{1-\rho} + E_t \left[ \sum_{j=0}^{\infty} \rho^j \left[ (1-\rho) d_{t+1+j} - r_{t+1+j} \right] \right], \tag{1}$$

   where $p_t = \log P_t$ is a log-stock price, $d_t = \log D_t$ is the corresponding log-dividend, $r_t = \log([P_t + D_t]/P_{t-1})$ is the log-return, and $k$ and $\rho$ are constants, with $0 < \rho < 1$.

   (a) **[10]** The equation (1) is obtained as a solution to the forward difference equation

$$p_t = \rho p_{t+1} + k + (1-\rho) d_{t+1} - r_{t+1}. \tag{2}$$

   Show how equation (2) is obtained from a log-linearization of the definition of $r_t$.

   (b) **[5]** Explain why the model (1) is more general than the linear present value / Gordon constant growth model.

   (c) **[10]** One way to test the constant growth model is via long-horizon regressions of the form.

$$r_{t+K,K} := r_{t+1} + \ldots + r_{t+K} = \alpha(K) + \beta(K)(d_t - p_t) + \eta_{t+K,K},$$

   where $\alpha(K)$ and $\beta(K)$ are regression coefficients, depending on $K$, and $\eta_{t+K,K}$ is a disturbance term. Explain why $\eta_{t+K,K}$ will display autocorrelation if $K > 1$. How can one correct the regression standard errors for this type of autocorrelation?

5. Let $\mathbf{R}_{Nt}$ denote an $N$-vector of the monthly stock returns, and let $\mathbf{R}_{Kt}$ denote another $K$-vector of monthly stock returns. It is assumed that

$$\begin{pmatrix} \mathbf{R}_{Nt} \\ \mathbf{R}_{Kt} \end{pmatrix} \sim \text{NID}\left( \begin{pmatrix} \boldsymbol{\mu}_N \\ \boldsymbol{\mu}_K \end{pmatrix}, \begin{bmatrix} \boldsymbol{\Omega}_{NN} & \boldsymbol{\Omega}_{NK} \\ \boldsymbol{\Omega}_{KN} & \boldsymbol{\Omega}_{KK} \end{bmatrix} \right). \tag{3}$$

An investor has a portfolio consisting of the last $K$ stocks, and wants to test if there are diversification benefits to adding some of the other $N$ stocks to his portfolio. For this purpose he collects a sample of monthly data $\{\mathbf{R}_{Nt}, \mathbf{R}_{Kt}, t = 1, \ldots, T\}$, and estimates, by least-squares, the multivariate regression

$$\mathbf{R}_{Nt} = \mathbf{a} + \mathbf{B}\mathbf{R}_{Kt} + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T. \tag{4}$$

(a) **[5]** What are the true (population) values of $\mathbf{a}$ and $\mathbf{B}$, implied by (3)?

(b) **[10]** The *intersection* hypothesis is $H_0 : \mathbf{a} = (\boldsymbol{\iota} - \mathbf{B}\boldsymbol{\iota})\gamma_0$, where $\boldsymbol{\iota}$ is a vector of ones, and $\gamma_0$ is a zero-beta return (a free parameter). Explain this hypothesis, and indicate how it can be tested.

(c) **[10]** Suppose now that a risk-free asset is available, paying a constant interest rate $R_f$. Let $\mathbf{Z}_{Nt} = \mathbf{R}_{Nt} - R_f$ and $\mathbf{Z}_{Kt} = \mathbf{R}_{Kt} - R_f$, the excess returns. Letting $\hat{\mathbf{a}}$ denote the estimated intercept in a least-squares regression $\mathbf{Z}_{Nt} = \mathbf{a} + \mathbf{B}\mathbf{Z}_{Kt} + \boldsymbol{\varepsilon}_t$, show that

$$\hat{\mathbf{a}}|\mathbf{X} \sim N\left( \boldsymbol{\alpha}, \frac{1}{T}\left[ 1 + \hat{\boldsymbol{\mu}}_K' \hat{\boldsymbol{\Omega}}_{KK}^{-1} \boldsymbol{\mu}_K \right] \boldsymbol{\Sigma} \right),$$

where $\mathbf{X}$ is the regressor matrix, where $\hat{\boldsymbol{\mu}}_K$ and $\hat{\boldsymbol{\Omega}}_{KK}$ are the sample mean vector and variance matrix of $\mathbf{R}_{Kt}$, and where $\boldsymbol{\Sigma}$ is the variance matrix of $\boldsymbol{\varepsilon}_t$.

6. Let $y_t = (y_{1t}, y_{2t})'$ be a vector of daily returns on 2 different assets, which is assumed to satisfy $y_t|\Omega_{t-1} \sim N(0, H_t)$, where

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix},$$

the $2 \times 2$ conditional covariance matrix of $y_t$. It is assumed that $H_t$ follows a DCC specification:

$$\begin{aligned} h_{11,t} &= \omega_1 + \alpha_1 y_{1,t-1}^2 + \beta_1 h_{11,t-1}, \\ h_{22,t} &= \omega_2 + \alpha_2 y_{2,t-1}^2 + \beta_2 h_{22,t-1}, \\ h_{12,t} &= \rho_{12,t}\sqrt{h_{11,t}h_{22,t}}, \end{aligned}$$

where $\rho_{12,t} = q_{12,t}/\sqrt{q_{11,t}q_{22,t}}$, and

$$\begin{aligned} q_{ij,t} &= \bar{q}_{ij}(1 - \alpha - \beta) + \alpha u_{i,t-1}u_{j,t-1} + \beta q_{ij,t-1}, \qquad i,j = 1,2, \\ u_{it} &= y_{it}/\sqrt{h_{ii,t}}, \qquad i = 1,2. \end{aligned}$$

(a) **[10]** What are the main advantages and limitations of the DCC model?

(b) **[5]** Decribe how the parameters of the model can be estimated.

(c) **[10]** Describe how the correlation specification may be tested, using the residuals $\hat{u}_{1t}$ and $\hat{u}_{2t}$, and the estimated correlation $\hat{\rho}_{12,t}$