Master of Econometrics - Game Theory

08-01-2010, from 14.00 till 17.00 hours.

Exercise 1 A correct and well motivated answer to each of the following problems scores 15 points.

a *Auctions*

15 Explain and justify the following claims.

- A *first* price sealed bid auction is equivalent with a *descending* (Dutch) auction.
- A *second* price sealed bid auction is related to but not quite equivalent with an *ascending* sequential (English) auction. Explain the essential differences.
- In a *public value* auctions participants tend to bid too much (the *winner's curse*).

b *Equilibria of Strategic Games*

15 Let G be a finite strategic game with the set $A = A_1 \times ... \times A_n$ of action profiles.

- Define (in words or formulas) a *strategic* equilibrium of G in mixed strategies.
- Sketch the idea of the proof of existence of such an equilibrium (Nash).
- Define (in words or formulas) a *correlated* equilibrium of G (Aumann). You may use A as the space of states.

c *Sequential and Trembling Hand Perfect Equilibria*

15 Let Γ be a finite extensive game with *imperfect* information.

- Give a precise definition of a *weakly sequential* equilibrium of Γ.
- A *sequential* equilibrium must have an additional property. What is it?
- Explain how a *trembling hand perfect* equilibrium can be used in order to prove that such a sequential equilibrium always exists.

d *Signaling and Screening*

15 Let Γ be a signaling game with n types of the sender who can send one of m signals, and k actions for the receiver, each of which can be played after every signal.

- How many pure strategies have both players in the strategic form of Γ?
- How many players are there in the agent strategic form of Γ?

In the corresponding *screening* game the sender commits himself in advance openly to one of his, possible mixed, strategies available to him in the signaling game.

- Describe, in words or symbols, the spaces of strategies of both players in the screening game with the same types, signals and actions as in Γ.

Exercise 2 (30) *Auctions*

a Formulate the Revenue Equivalence Theorem in its most general form (known to you).

Consider two risk averse bidders with the vNM utility function $u(x) = x^c$, $0 < c < 1$ and private valuations distributed independently and uniformly on $[0, 1]$.

b What auction format, the first price or the second price, would you choose as a risk neutral seller? (reason by a comparison with the case of risk neutral bidders).

c Derive the differential equation for the symmetric equilibrium bidding function $b(v)$ in the case of two risk averse bidders. Show that the solutions of this equation are

$$b(v) = \frac{v}{1+c} + \frac{a}{v^{1/c}}, \quad \text{with some constant } a \geq 0.$$

d The seller can increase her expected revenue by excluding bidders with low valuations.

Determine a as a function of the imposed reservation price r.

e The value of r maximizing the expected profit of the seller will depend on c.

Should you raise or lower r as the bidders are becoming increasingly risk averse?

f Calculate the optimal value of r for the case of the risk neutral bidders, $c = 1$, and that for the case of the risk averse bidders with $c = \frac{1}{2}$.

Exercise 3 (30) *Signaling Games*

The incumbent firm $\boldsymbol{A}$ is threatened by the entry of firm $\boldsymbol{B}$. The financial situation of $\boldsymbol{A}$ is strong - type $\boldsymbol{S}$, with probability ρ, or weak - type $\boldsymbol{W}$, with probability 1-ρ.

$\boldsymbol{A}$ can report her situation as good - signal **g**, or bad - signal **b**. $\boldsymbol{B}$ is not able to check if $\boldsymbol{A}$'s report can be trusted. Eventually $\boldsymbol{A}$ will have to pay 1 for a false report.

In any case $\boldsymbol{A}$ earns 2 if ~~C~~ B quits - **out**, and loses 1 if $\boldsymbol{B}$ enters - **in**. $\boldsymbol{B}$ earns 0 if he stays **out**. After the entry he gains 1 if $\boldsymbol{A}$ is weak, and loses 1 if $\boldsymbol{A}$ is strong.

$\boldsymbol{A}$ and $\boldsymbol{B}$ are playing a sequential equilibrium of this signaling game.

a Prove that there is no separating equilibrium, for any value of ρ.

b $\rho = \frac{3}{4}$. Construct two pooling equilibria.

Which of them is unreasonable? Provide a plausible argument.

c $\rho = \frac{1}{4}$. Prove that in this case there is no pooling equilibrium either.

Construct a hybrid equilibrium in mixed strategies.

Provide a full specification of this (weakly) sequential equilibrium.