Game Theory – 21-01-2005, 14.00-17.00.

Problem 1 Consider the following symmetric game G

	A	B
A	$2-c,\ 2-c$	$0,\ \ 2$
B	$2,\ \ 0$	$-c,\ -c$

, with $c \in \mathbb{R}$.

a Find all equilibria of G. There are three cases here, $c < 0$, $c = 0$ and $c > 0$.

b Which of these equilibria are *trembling hand perfect*? and why?

c Which strategies are *evolutionary stable* if $c < 0$, which if $c > 0$?

d Find for $c > 0$ the *correlated equilibrium* of G which maximizes the sum of expected payoffs of both players. There are two cases, $c \leq 1$ and $c \geq 1$. You may assume that it is symmetric and that (B, B) has probability 0.

Problem 2 Seller owns a car which depending on its quality is worth 0 or 3 to her. Buyer believes that the car is *good* or *bad* with the probability of $\frac{1}{2}$. After the purchase it is worth 1 or 4 to him. Seller can demand a low price $\ell \leq 1$, or a high price $1 < h \leq 4$. Buyer will buy if the price is low.

a A weakly sequential equilibrium in pure strategies exists only for specific values of ℓ and h. Construct this equilibrium. Argue carefully.

b For any given $3 \leq h^* \leq 4$ there exists an equilibrium in mixed strategies where Seller of a good car asks h^*. For such an equilibrium you need three probabilities, all depending on h^*. What do they mean? What are they?

c Given that Seller moves first, which of the equilibria found in **b** is most convincing? Asuming still that **B**uyer buys for sure if the price is 1, what kind of offer would **B**uyer make to Seller if he was able to move first?

Problem 3 Consider an auction with independent and discrete valuations. For simplicity assume that there are two buyers and that each of them, with equal probability, can have a low valuation $\ell = 1$, or a high valuation $h = 2$.

a In the *first price* auction a low value buyer will bid 1, and that a high value buyer will randomize over an interval of bids $(1, \overline{b}]$. Find $\overline{b}$, and the distribution function of the bids of such buyer (making use of the fact that the expected surplus of all bids in $(1, \overline{b}]$ must be the same).

b In the *all pay* auction all buyers will randomize. Argue that it is not possible that bidders of both types are indifferent for the same bids $b_1 \neq b_2$. Hence, in an equilibrium there are values $\underline{b}$ and $\overline{b}$ such that a low value bidder will randomize over $[0, \underline{b}]$ and the high value bidder over $[\underline{b}, \overline{b}]$. Why? Now you can find the distribution functions of the bids of both types.