

Caput Derivatives: October 30, 2003

Exam + Answers

Total time: 2 hours and 30 minutes.

Note 1: You are allowed to use books, course notes, and a calculator.

Question 1. [20 points]

Consider an investor who wants to trade in a 6-month forward contract on a non-dividend-paying stock. Current stock price is \$30. There is a difference between borrowing and lending rates. Suppose that the investor can borrow at annual rates of 6% and lend at 4.5% (continuously compounded). For which delivery prices of the forward contract can the investor make an arbitrage profit? Describe the arbitrage strategy.

Arbitrage if $F < 30.68$ or if $F > 30.91$:

*If $F < 30.68 = 30 * \exp(0.045 * 0.05)$: short equity (and put proceeds in money market account at 4.5%), and long position in forward*

*If $F > 30.68 = 30 * \exp(0.045 * 0.05)$: buy equity (financed by borrowing at 6%), and short position in forward*

Question 2. [30 points]

Consider a binomial firm value model. A firm is currently worth \$6,000,000. In one year from now, the firm value will either be \$7,000,000 or \$5,000,000. The actual upward probability equals 70%. The firm has issued equity and zero-coupon corporate bonds with one-year maturity, and the total face value of the bonds is \$5,500,000. The one-year default-free interest rate (i.e., the interest rate associated with a default-free money-market account) equals 4% (continuously compounded).

- a) Determine the current value of equity and of the zero-coupon bonds.

*Use risk-neutral valuation: risk-neutral upward probability is $0.6224 = (\exp(rT) - d) / (u - d)$.
Current bond value is $(0.6224 * 5,500,000 + (1 - 0.6224) * 5,000,000) / \exp(0.04) = 5,102,945$
Current equity value is $(0.6224 * 1,500,000 + (1 - 0.6224) * 0) / \exp(0.04) = 897,055$
Bond plus equity value equals 6 million as it should be.*

- b) Calculate the expected one-year return on equity and on the zero-coupon bonds. Interpret the difference between these expected returns and the risk-free rate.

*Bond: Exp return = $(0.7 * 5,500,000 + 0.3 * 5,000,000) / 5,102,945 - 1$
or continuous compounding: $\ln((0.7 * 5,500,000 + 0.3 * 5,000,000) / 5,102,945) = 0.047$*

*Equity: Exp return = $(0.7 * 1,500,000 + 0.3 * 0) / 897,055 - 1$
or continuous compounding: $\ln((0.7 * 1,500,000 + 0.3 * 0) / 897,055) = 0.16$*

Both expected returns are higher than risk-free rate, implying that investors are compensated for the risk they take. Equity has the largest risk and therefore the highest expected return. This is caused by the difference between the risk-neutral and actual probabilities.

- c) Suppose an investor has bought a one-year zero-coupon bond of this firm with face value \$1000. The investor wants to hedge the associated risk by trading in the firm's equity and a money market account. Show how the investor can hedge the corporate bond risk, by calculating the positions in equity and the money market account.

Consider a 1\$ investment in equity. The payoff of equity in upper state is $1.500.000/897.055 = \$1.67$, and 0\$ in downstate.

Consider a portfolio in the bond and investing \$w dollars in equity. Payoff in upper state equals $1000 + w*1.67$, and 909.1 in downstate ($909.1 = 1000 * (5.000.000/5.500.000)$). Riskless portfolio if $1000 + w*.167 = 909.1$, so $w = -54.43$. So short \$54.43 equity.

Question 3 [20 points]

For options on equity indices the pattern of implied volatilities (IV) across strike prices tends to be downward sloping, i.e. high IVs for low strikes and low IVs for high strikes. Give two option-pricing models that imply such a decreasing pattern of implied volatilities, and explain briefly for each model how the decreasing pattern is obtained.

Jump-diffusion model with downward jumps only, CEV model with $\alpha < 1$, Stochastic volatility model with negative correlation between equity and volatility, Merton model with equity as option on firm value, implied tree model that fits observed options perfectly.

Question 4 [30 Points]

Consider a long position in the strategies listed below. Indicate for each strategy whether respectively the delta, gamma, vega, and theta is (i) (close to) zero, (ii) larger than zero, (iii) smaller than zero, or (iv) it is not possible to indicate the sign. You do not have to numerically calculate the greeks. Motivate your answer!

- a) Strip (using ATM put and call options)

Strip has long position in 2 ATM puts and 1 ATM call

Delta is negative: ATM options have delta around 0.5, given 2 put options vs 1 call, the total delta is negative.

Gamma and vega are positive: long position in put or call always has positive gamma and vega, so total gamma and vega is positive.

Theta is negative: long position in put or call (almost) always has negative theta, so total theta is negative.

- b) Butterfly spread (using an OTM, ATM and ITM call option)

Butterfly spread: buy ITM call, sell 2 ATM calls, and buy OTM call.

Delta is close to zero: Long ITM and OTM delta roughly add up to one, and short 2 ATM calls delta's add up to -1 , total is around zero.

Gamma and vega are negative: ATM options have largest gamma and vega, and given short position in these options, total gamma and vega are negative.

Theta is positive: Intuitively, if time goes by (everything else equal), more chance that butterfly spread generates positive payoff.

Caput Derivatives: November, 2004

Exam + Answers

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Question 1. [20 points]

Suppose that the Euro/dollar exchange rate is currently 1.30 dollar per Euro. The 3-month forward Euro/dollar rate is equal to 1.31 dollar per Euro.

What information does the difference between the forward Euro/dollar rate and the current Euro/dollar rate give about the expected future development of the Euro/dollar rate? Explain your answer.

The difference between the forward and spot currency rate is completely determined by the maturity and the difference between the domestic and foreign interest rate. Therefore, it does not directly reflect any market expectations on the development of the currency rate. Only if there is no risk premium on currency risk, the forward rate equals the expected currency rate (see section 3.15 of Hull)

Question 2. [15 points]

Give an intuitive explanation of why early exercise of an American put option becomes more attractive as the risk-free rate increases and volatility decreases.

The advantage of early exercise is that a payoff is received now instead of at maturity. This advantage becomes larger if the risk-free rate is larger. A disadvantage of early exercise is that one does not profit from further decreases in the stock price. This is less likely if volatility is low. In the extreme case, if volatility approaches zero, it is always optimal to exercise immediately if the option is in the money.

Question 3. [20 points]

A trader is using the Black-Scholes model to delta-hedge a long position in an equity put option. She uses the underlying equity to delta-hedge and rebalances her position once per day. The underlying equity price changes continuously over time according to the Black-Scholes model.

Given this hedging strategy, would she prefer a small or large movement in the equity price on a given day? Explain your answer.

See figure 14.8 for the payoff in case of discrete delta-hedging. Given the positive gamma of a long put, a profit is obtained in case of large movements (positive or negative), and a loss in case of small movements. The trader would thus prefer large movements. If transaction costs are large, this profit/loss has to be compared with the costs of rebalancing.

Question 4. [25 points]

a) Suppose the expected return on a stock equals 12% per year, while the risk-free rate equals 4%. Consider a call option on this stock. Will the expected return on this call option be (i) larger than 12%, (ii) equal to 12%, (iii) between 4% and 12%, (iv) equal to 4%, or (v) smaller than 4%? (All numbers are annualized)

Motivate your answer!

Larger than 12%. Given that the call option is positively related to the stock price, and has more extreme and volatile returns, the expected call return will be larger than the expected stock return (in a sense, the call has a higher 'beta'). This can also be seen from the formula $e^{mT} = e^{rT}[qf_u + (1-q)f_d] / [pf_u + (1-p)f_d]$ (as given on slides of lecture 2).

b) Answer the same question as in a), now in case the expected return on the stock equals 4%.

Again motivate your answer!

If the stock has expected return equal to the risk-free rate, investors are apparently risk-neutral. In that case, a call option will also have expected return equal to the risk-free rate. Again, the formula $e^{mT} = e^{rT}[qf_u + (1-q)f_d] / [pf_u + (1-p)f_d]$ shows this.

Question 5. [20 points]

Consider the rating migration model, with constant recovery rate and constant default-free interest rates. Discuss intuitively (without calculations) how one would value a 10-year credit default swap (with annual payments) using this model.

Each year, the CDS pays out the loss on a reference corporate bond (approximately 1-recovery rate), if the bond defaults in that year. The first step is calculate risk-neutral default probabilities for each year, conditional upon surviving until the year before. These multi-year default probabilities can be obtained using the historical migration matrix, and correcting it to get a risk-neutral migration matrix. A fixed swap rate is paid until default. Now one can solve for this swap rate (the CDS rate), by equating the risk-neutral expected value of the payoff due to default to the risk-neutral expected value of the swap rate payments.

Alternatively, one can get the CDS rate as the difference between the corporate bond yield and government bond yield. Now, the corporate bond has to be priced using the rating migration model (again calculated the risk-neutral expected payoff of the bond, incorporating the default probabilities). Note that the bond has coupon payments, which need to be incorporated as well.