Caput Derivatives: November, 2004

Exam + Answers

Total time: 2 hours and 30 minutes.
Note: You are allowed to use books, course notes, and a calculator.

Question 1. [20 points]

Suppose that the Euro/dollar exchange rate is currently 1.30 dollar per Euro. The 3-month forward Euro/dollar rate is equal to 1.31 dollar per Euro.

What information does the difference between the forward Euro/dollar rate and the current Euro/dollar rate give about the expected future development of the Euro/dollar rate? Explain your answer.

The difference between the forward and spot currency rate is completely determined by the maturity and the difference between the domestic and foreign interest rate. Therefore, it does not directly reflect any market expectations on the development of the currency rate. Only if there is no risk premium on currency risk, the forward rate equals the expected currency rate (see section 3.15 of Hull)

Question 2. [15 points]

Give an intuitive explanation of why early exercise of an American put option becomes more attractive as the risk-free rate increases and volatility decreases.

The advantage of early exercise is that a payoff is received now instead of at maturity. This advantage becomes larger if the risk-free rate is larger. A disadvantage of early exercise is that one does not profit from further decreases in the stock price. This is less likely if volatility is low. In the extreme case, if volatility approaches zero, it is always optimal to exercise immediately if the option is in the money.

Question 3. [20 points]

A trader is using the Black-Scholes model to delta-hedge a long position in an equity put option. She uses the underlying equity to delta-hedge and rebalances her position once per day. The underlying equity price changes continuously over time according to the Black-Scholes model.

Given this hedging strategy, would she prefer a small or large movement in the equity price on a given day? Explain your answer.

See figure 14.8 for the payoff in case of discrete delta-hedging. Given the positive gamma of a long put, a profit is obtained in case of large movements (positive or negative), and a loss in case of small movements. The trader would thus prefer large movements. If transaction costs are large, this profit/loss has to be compared with the costs of rebalancing.

Question 4. [25 points]

a) Suppose the expected return on a stock equals 12% per year, while the risk-free rate equals 4%. Consider a call option on this stock. Will the expected return on this call option be (i) larger than 12%, (ii) equal to 12%, (iii) between 4% and 12%, (iv) equal to 4%, or (v) smaller than 4%? (All number are annualized)

Motivate your answer!

Larger than 12%. Given that the call option is positively related to the stock price, and has more extreme and volatile returns, the expected call return will be larger than the expected stock return (in a sense, the call has a higher 'beta'). This can also be seen from the formula $e^{mT} = e^{rT}[qf_u + (1–q)\, f_d] / [pf_u + (1–p)\, f_d]$ *(as given on slides of lecture 2).*

b) Answer the same question as in a), now in case the expected return on the stock equals 4%.

Again motivate your answer!

If the stock has expected return equal to the risk-free rate, investors are apparently risk-neutral. In that case, a call option will also have expected return equal to the risk-free rate. Again, the formula $e^{mT} = e^{rT}[qf_u + (1–q)\, f_d] / [pf_u + (1–p)\, f_d]$ *shows this.*

Question 5. [20 points]

Consider the rating migration model, with constant recovery rate and constant default-free interest rates. Discuss intuitively (without calculations) how one would value a 10-year credit default swap (with annual payments) using this model.

Each year, the CDS pays out the loss on a reference corporate bond (approximately 1-recovery rate), if the bond defaults in that year. The first step is calculate risk-neutral default probabilities for each year, conditional upon surviving until the year before. These multi-year default probabilities can be obtained using the historical migration matrix, and correcting it to get a risk-neutral migration matrix. A fixed swap rate is paid until default. Now one can solve for this swap rate (the CDS rate), by equating the risk-neutral expected value of the payoff due to default to the risk-neutral expected value of the swap rate payments.

Alternatively, one can get the CDS rate as the difference between the corporate bond yield and government bond yield. Now, the corporate bond has to priced using the rating migration model (again calculated the risk-neutral expected payoff of the bond, incorporating the default probabilities). Note that the bond has coupon payments, which need to be incorporated as well.