Caput Derivatives: October 30, 2003

Exam + Answers

Total time: 2 hours and 30 minutes.

Note 1: You are allowed to use books, course notes, and a calculator.

Question 1. [20 points]

Consider an investor who wants to trade in a 6-month forward contract on a non-dividend-paying stock. Current stock price is $30. There is a difference between borrowing and lending rates. Suppose that the investor can borrow at annual rates of 6% and lend at 4.5% (continuously compounded). For which delivery prices of the forward contract can the investor make an arbitrage profit? Describe the arbitrage strategy.

Arbitrage if F < 30.68 or if F > 30.91:

*If F < 30.68 = 30*exp(0.045*0.05): short equity (and put proceeds in money market account at 4.5%), and long position in forward*

*If F > 30.68 = 30*exp(0.045*0.05): buy equity (financed by borrowing at 6%), and short position in forward*

Question 2. [30 points]

Consider a binomial firm value model. A firm is currently worth $6.000.000. In one year from now, the firm value will either be $7.000.000 or $5.000.000. The actual upward probability equals 70%. The firm has issued equity and zero-coupon corporate bonds with one-year maturity, and the total face value of the bonds is $5.500.000. The one-year default-free interest rate (i.e., the interest rate associated with a default-free money-market account) equals 4% (continuously compounded).

a) Determine the current value of equity and of the zero-coupon bonds.

Use risk-neutral valuation: risk-neutral upward probability is 0.6224 = (exp(rT)-d)/(u-d).
*Current bond value is (0.6224*5.500.000 + (1-0.6224)*5.000.000)/exp(0.04) = 5.102.945*
*Current equity value is (0.6224*1.500.000 + (1-0.6224)*0)/exp(0.04) = 897.055*
Bond plus equity value equals 6 million as it should be.

b) Calculate the expected one-year return on equity and on the zero-coupon bonds. Interpret the difference between these expected returns and the risk-free rate.

*Bond: Exp return = (0.7*5.500.000 + 0.3 * 5.000.000)/5.102.945 – 1*
*or continuous compounding: ln (0.7*5.500.000 + 0.3 * 5.000.000)/5.102.945) = 0.047*

*Equity: Exp return = (0.7*1.500.000 + 0.3 * 0)/897.055 – 1*
*or continuous compounding: ln (0.7*1.500.000 + 0.3 * 0)/897.055) = 0.16*

Both expected returns are higher than risk-free rate, implying that investors are compensated for the risk they take. Equity has the largest risk and therefore the highest expected return. This is caused by the difference between the risk-neutral and actual probabilities.

c) Suppose an investor has bought a one-year zero-coupon bond of this firm with face value $1000. The investor wants to hedge the associated risk by trading in the firm's equity and a money market account. Show how the investor can hedge the corporate bond risk, by calculating the positions in equity and the money market account.

Consider a 1$ investment in equity. The payoff of equity in upper state is 1.500.000/897.055 = $1.67, and 0$ in downstate.

*Consider a portfolio in the bond and investing $w dollars in equity. Payoff in upper state equals 1000 + w*1.67, and 909.1 in downstate (909.1 = 1000 * (5.000.000/5.500.000)). Riskless portfolio if 1000 + w*.167 = 909.1, so w = -54.43. So short $54.43 equity.*

Question 3 [20 points]

For options on equity indices the pattern of implied volatilities (IV) across strike prices tends to be downward sloping, i.e. high IVs for low strikes and low IVs for high strikes. Give two option-pricing models that imply such a decreasing pattern of implied volatilities, and explain briefly for each model how the decreasing pattern is obtained.

Jump-diffusion model with downward jumps only, CEV model with alpha<1, Stochastic volatility model with negative correlation between equity and volatility, Merton model with equity as option on firm value, implied tree model that fits observed options perfectly.

Question 4 [30 Points]

Consider a long position in the strategies listed below. Indicate for each strategy whether respectively the delta, gamma, vega, and theta is (i) (close to) zero, (ii) larger than zero, (iii) smaller than zero, or (iv) it is not possible to indicate the sign. You do not have to numerically calculate the greeks. Motivate your answer!

a) Strip (using ATM put and call options)

Strip has long position in 2 ATM puts and 1 ATM call

Delta is negative: ATM options have delta around 0.5, given 2 put options vs 1 call, the total delta is negative.
Gamma and vega are positive: long position in put or call always has positive gamma and vega, so total gamma and vega is positive.
Theta is negative: long position in put or call (almost) always has negative theta, so total theta is negative.

b) Butterfly spread (using an OTM, ATM and ITM call option)

Butterfly spread: buy ITM call, sell 2 ATM calls, and buy OTM call.

Delta is close to zero: Long ITM and OTM delta roughly add up to one, and short 2 ATM calls delta's add up to –1, total is around zero.
Gamma and vega are negative: ATM options have largest gamma and vega, and given short position in these options, total gamma and vega are negative.
Theta is positive: Intuitively, if time goes by (everything else equal), more chance that butterfly spread generates positive payoff.