

Practice Exam (Solutions)

June 6, 2008

Course: Finance for AEO

Length: 2 hours

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Students are expected to conduct themselves properly during examinations and to obey any instructions given to them by examiners and invigilators. Firm action will be taken in the event that academic fraud is discovered.

Instructions

- There are five questions, each worth 10 points.
- Answer all five questions.
- The exam is closed book, no translation computers are allowed. You may use a dictionary.
- You are allowed to use a non-graphical calculator.
- Explain your answers. Partial credit is given for incomplete answers.
- Write clearly.

Question 1

Currently, there are three government bonds trading. All three are zero coupon bonds and have a face value of 1000, but they differ in maturity. Bond A matures in one year and trades for 952.381, bond B matures in two years and trades for 890, and bond C matures in three years and trades for 839.619.

- (a) Construct a yield curve showing the 1, 2, and 3-year interest rates.

$$r_1 = \frac{1000}{952.381} - 1 = 5\%, r_2 = \left(\frac{1000}{890}\right)^{\frac{1}{2}} - 1 = 6\%, r_3 = \left(\frac{1000}{839.619}\right)^{\frac{1}{3}} - 1 = 6\%$$

- (b) You are asked to value a newly issued government bond. This bond (bond D) is a three year coupon bond with a face value of 1000 and a coupon rate of 6%. What is the right price for bond D?

$$P = \frac{60}{1.05} + \frac{60}{1.06^2} + \frac{1060}{1.06^3} = 1000.539$$

- (c) If the one-year risk-free interest rate increases by 100 basis points, by how many percent would the price of bond D change?

A 100 basis point increase in the one year rate means that the one year yield increases from 5% to 6%. Therefore: $\tilde{P} = \frac{60}{1.06} + \frac{60}{1.06^2} + \frac{1060}{1.06^3} = 1000$. The bond price declines by 0.05%

Question 2

Clogger B.V., a chain of shoe stores, is considering expanding its current business and plans to open another Clogger Super Store. The project would bring in additional sales of 100 million Euro per year forever, starting in one year (at $t=1$), 88% of which will be spent on labor, administrative expenses etc. To implement the expansion, the company will have to carry extra inventory of 10% of sales and additional accounts receivable of 5% of sales. Both the extra inventory and the extra accounts receivable would be necessary at $t=1$. The expansion costs 70 million Euro now ($t=0$) and can be depreciated linearly over ten years starting at $t=1$.

Currently, the company has 100 million shares of common stock outstanding, trading at 3 Euro at the stock exchange and an equity beta of 3. Clogger B.V. also has 700 million Euro of debt on its balance sheet, and its debt has an expected rate of return of 5.9%.

To answer the questions below, make the following assumptions: The risk free rate of return is 5% per year, the market risk premium is 9% per year. The company does not pay corporate taxes and markets are perfect.

- (a) Calculate the beta of Clogger BV's assets

First, calculate the value of equity: $E = 100 \times 3 = \$300$ million. The value of the company's assets is therefore $V = E + D = 300 + 700 = \1000 million. The debt has an expected return of 5.9%. Since the SML has to hold for *any* asset, it has to be the case that

$$5.9\% = r_f + \beta_D(E(r_m) - r_f) \text{ and therefore the beta of debt is } \beta_D = \frac{5.9-5}{9} = 0.1$$

$$\text{Therefore the beta of the assets is } \beta_A = \frac{300}{1000}\beta_E + \frac{700}{1000}\beta_D = 0.97.$$

- (b) Calculate the expected return on Clogger BV's assets.

There are two ways to calculate the expected return on the company's assets (the pre-tax WACC):

(i) Plug the asset beta into the SML: $r_A = r_f + \beta_A(E(r_m) - r_f) = 5 + 0.97 \times 9 = 13.73\%$

(ii) Calculate the expected return of equity via the SML: $r_E = r_f + \beta_E(E(r_m) - r_f) = 5 + 3 \times 9 = 32\%$ and then calculate the weighted average cost of capital directly:

$$r_A = \frac{300}{1000}r_E + \frac{700}{1000}r_D = 0.3 \times 32 + 0.7 \times 5.9 = 13.73\%$$

- (c) Calculate the impact of the project on Clogger BV's unlevered net income in years 0 and 1.

Impact at $t = 0$: none

Impact at $t = 1$: Change in net income = Sales - Cost - Depreciation - Taxes = $100 - 88 - 7 - 0 = 5$ million Euro

(d) Calculate the impact of the project on Clogger BV's free cash flows in years 0 and 1

Calculate the impact on free cash flows as (net income + depreciation - change in net working capital - capital expenses)

Impact at $t = 0$: $0 + 0 - 0 - 70 = -70$ million Euro

Impact at $t = 1$: $5 + 7 - 15 - 0 = -3$ million Euro (Note that the change in working capital in year one is 15% of sales)

(e) Calculate the present value of the projects cash flows.

Note that in year $t = 1$ the FCF is -27 Euro, thereafter it will be $5 + 7 = 12$ million Euro. Therefore, $PV = \frac{-3}{1.1373} + \frac{12}{1.1373} = 74.21$. Note that we are discounting using the cost of capital of the firm.

(f) Calculate the NPV of the project. Should Clogger BV expand?

$NPV = -70 + 74.21 = 4.21$ million Euro. Since the NPV is greater than zero, the firm should expand.

(g) Does it matter how Clogger BV finances the expansion? Explain why or why not.

Since financial markets are perfect (especially since there are no corporate taxes!), according to the theorems by Modigliani and Miller, it does not matter how the project is financed.

Question 3

ABC corporation currently has 100 million shares outstanding, each trading at \$6 and debt valued \$620 million. Furthermore, its debt cost of capital is 6%, its equity cost of capital is 10%, and its marginal corporate tax rate is 40%. ABC also has cash of \$20 million and always keeps its D/E ratio constant.

Suppose ABC is considering the acquisition of another firm in its industry (i.e. the targeted firm is of similar risk). The acquisition is expected to increase ABC's (unlevered) free cash flow by \$5 million in the first year, and this contribution is expected to grow at a rate of 2% per year from then on. ABC has negotiated a purchase price of \$100 million.

ABC can either choose to finance the acquisition with 50 million of permanent debt or keep its D/E ratio at its current level after the acquisition.

- (a) Calculate ABC's D/E ratio and its D/Value ratio

$$E = 600 \text{ and net debt } D = 620 - 20 = 600, \text{ therefore } D/E = 1 \text{ and } D/(D+E) = 0.5$$

- (b) Calculate ABC's pre-tax WACC

$$\text{pre-tax WACC} = 0.5 \times 10 + 0.5 \times 6 = 8\%$$

- (c) Calculate ABC's after-tax WACC

$$\text{after-tax WACC} = 0.5 \times 10 + 0.5 \times 6 \times (1 - 0.4) = 6.8\%$$

- (d) Calculate the NPV of the project if ABC maintains a fixed D/E ratio (equal to its current one) after the acquisition.

$$\text{NPV} = V^L - 100 = \frac{5}{0.068 - 0.02} - 100 = \$4.17 \text{ million. (Here we use the "WACC method.")}$$

- (e) Using APV, calculate the NPV of the project if it is financed using 50 million of permanent debt.

$$\text{APV} = V^U + \text{PV}(\text{tax shield}) - \text{capital expenditure} = \frac{5}{0.10 - 0.02} + 50 \times 0.4 - 100 = \$3.33 \text{ million. Note that we estimate the PV without debt financing using the pre-tax WACC (which is equal to the cost of capital of the unlevered assets, } r_U \text{ in the book).}$$

Question 4

The current price of W Corporation stock is \$7. In each of the next two years, this stock price can either go up by \$2.50 or go down by \$2.00. The stock pays no dividends. The one-year risk-free interest rate is 3% and will remain constant.

(a) Draw the event tree. At $t=2$, what are the possible values the stock price can take on?

At $t = 1$: $P_1 = 9.5$ or $P_1 = 5$.
 At $t = 2$: $P_2 = 12$ or $P_2 = 7.5$ or $P_2 = 3$

(b) Calculate the price of a two-year European call option on W stock with an exercise price of \$7.

For the solution, I use risk-neutral valuation. You could also have used the binomial model.

For the first year, the risk free probability of an increase in the stock price is $p = 0.4911$
 At this value, $P_0 = 7 = \frac{1}{1.03} (p \times 9.5 + (1 - p) \times 5)$.

For the second year, there are two risk free probabilities:

- (i) $q_1 = 0.5077$ (such that $P_1 = 9.5 = \frac{1}{1.03} (q_1 \times 12 + (1 - q_1) \times 7.5)$)
- (ii) $q_2 = 0.478$ (such that $P_1 = 5 = \frac{1}{1.03} (q_2 \times 7.5 + (1 - q_2) \times 3)$).

The terminal values of the tree have the following probabilities and call payoffs:

	Probability	Payoff
up/up	pq_1	5
up/down	$p(1 - q_1)$	0.5
down/up	$(1 - p)q_2$	0.5
down/down	$(1 - p)(1 - q_2)$	0

Therefore the value of the call is:

$$C = \frac{1}{1.03^2} [pq_1 \times 5 + (p(1 - q_1) + (1 - p)q_2) \times 0.5 + (1 - p)(1 - q_2) \times 0] = 1.4038.$$

(c) Using put-call parity, calculate the price of a two-year European put option on W stock with an exercise price of \$7.

Put-call parity states that $C + PV(K) = P + S$ and therefore $P = 1.4038 + \frac{7}{1.03^2} - 7 = 1.002$

Question 5

- (a) Explain the over-investment problem of excessive leverage using an example.

Suppose a firm has to pay back debt worth 100 at the end of the year and currently has 90 in cash. If the firm does not do anything, it will default, the debt holders will get 90 and the equity holders nothing. Suppose instead the firm invests the 90 in a lottery ticket that pays off 1000 with a probability of 10% and zero otherwise. The equity holders would now get an expected payoff of $0.1 \times (1000 - 100) = 90$ which is larger than what they would get if they did not do this bad project.

- (b) The book value of a company's assets usually does not equal the market value of those assets. Explain in a few sentences some reasons for this difference.

Book values reflect historical costs of acquiring assets minus depreciation. If a firm makes good investments, the value of the assets is equal to the PV of future free cash flows, which is higher than costs. Book values usually also don't include intangible assets, such as patents that were developed within the firm or brand names, and ignore the value of tax shields.

- (c) Why does the expected return of a risky security generally differ from the risk-free interest rate? What determines the size of its risk premium? Explain in a few sentences.

The expected return of a risky security depends on its riskiness, which usually differs from the riskiness of risk-free government securities. In the CAPM, the risk premium depends on the market risk of the security, measured by its beta.

- (d) You own a stock market portfolio that has a beta of 2.4, but you are getting married to someone who has a portfolio with a beta of 0.4. You are three times as wealthy as your future husband/wife. What is the beta of your future joint portfolio?

$$\beta_{\text{married}} = w_{\text{you}}\beta_{\text{you}} + w_{\text{partner}}\beta_{\text{partner}} = 0.75 \times 2.4 + 0.25 \times 0.4 = 1.9$$