FACULTEIT ECONOMIE EN BEDRIJFSKUNDE
Afdeling Kwantitatieve Economie

Opgave 1

(a) Er geldt: $\overrightarrow{v} = \overrightarrow{OP} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ en $\overrightarrow{u} = \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$. Dus $\cos\theta = \dfrac{\overrightarrow{u}\bullet\overrightarrow{v}}{\|\overrightarrow{u}\|\cdot\|\overrightarrow{v}\|} = \dfrac{2\cdot 4 - 2\cdot 3}{\sqrt{4^2+3^2}\cdot\sqrt{2^2+(-2)^2}} = \dfrac{2}{5\cdot 2\sqrt{2}} = \dfrac{\sqrt{2}}{10}$.

(b) De gevraagde afstand is

$$\begin{aligned}\sqrt{\|\overrightarrow{u}-\overrightarrow{v}\|^2} &= \sqrt{(\overrightarrow{u}-\overrightarrow{v})\bullet(\overrightarrow{u}-\overrightarrow{v})} \\ &= \sqrt{\|\overrightarrow{u}\|^2 - 2\overrightarrow{u}\bullet\overrightarrow{v} + \|\overrightarrow{v}\|^2} \\ &= \sqrt{25-4+8} = \sqrt{29}.\end{aligned}$$

Opgave 2

(a) De normaalvorm is $\overrightarrow{n}\bullet\overrightarrow{x} = \overrightarrow{n}\bullet\overrightarrow{p}$ dus $\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = 12$. De algemene vorm is dus $2x + 4y + 4z = 12$.

(b) Zij $\overrightarrow{w} = \overrightarrow{AB} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$. Dan is $\text{proj}_{\overrightarrow{n}}(\overrightarrow{AB}) = \dfrac{\overrightarrow{n}\bullet\overrightarrow{w}}{\overrightarrow{n}\bullet\overrightarrow{n}}\overrightarrow{n} = \dfrac{-4}{36}\overrightarrow{n} = -\dfrac{1}{9}\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$. De gevraagde afstand is de lengte van deze vector, dus $\dfrac{1}{9}\sqrt{2^2+4^2+4^2} = \dfrac{6}{9} = \dfrac{2}{3}$.

Opgave 3

Er geldt:

$$\begin{pmatrix} 1 & -2 & 1 & 0 \\ 1 & -1 & -3 & 4 \\ -4 & 5 & 9 & -9 \end{pmatrix} \overset{II-I \text{ en } III+4I}{\Longleftrightarrow} \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & -9 \end{pmatrix}$$

$$\overset{III+3II}{\Longleftrightarrow} \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\overset{II+4III \text{ en } I-III}{\Longleftrightarrow} \begin{pmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\overset{I+2II}{\Longleftrightarrow} \begin{pmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

Dus $x = 29$, $y = 16$ en $z = 3$.